

# D mesons and charmonium states in asymmetric nuclear matter at finite temperatures

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## Abstract

We investigate the in-medium masses of  $D$  and  $\bar{D}$  mesons in the isospin-asymmetric nuclear matter at finite temperatures arising due to the interactions with the nucleons, the scalar isoscalar meson  $\sigma$ , and the scalar iso-vector meson  $\delta$  within a SU(4) model. However, since the chiral symmetry is explicitly broken for the SU(4) case due to the large charm quark mass, we use the SU(4) symmetry here only to obtain the interactions of the  $D$  and  $\bar{D}$  mesons with the light hadron sector, but use the observed values of the heavy hadron masses and empirical values of the decay constants. The in-medium masses of  $J/\psi$  and the excited charmonium states ( $\psi(3686)$  and  $\psi(3770)$ ) are also calculated in the hot isospin asymmetric nuclear matter in the present investigation. These mass modifications arise due to the interaction of the charmonium states with the gluon condensates of QCD, simulated by a scalar dilaton field introduced to incorporate the broken scale invariance of QCD within the effective chiral model. The change in the mass of  $J/\psi$  in the nuclear matter with the density is seen to be rather small, as has been shown in the literature by using various approaches, whereas, the masses of the excited states of charmonium ( $\psi(3686)$  and  $\psi(3770)$ ) are seen to have considerable drop at high densities. The present study of the in-medium masses of  $D$  ( $\bar{D}$ ) mesons as well as of the charmonium states will be of relevance for the observables from the compressed baryonic matter, like the production and collective flow of the  $D$  ( $\bar{D}$ ) mesons, resulting from the asymmetric heavy ion collision experiments planned at the future facility of the FAIR, GSI. The mass modifications of  $D$  and  $\bar{D}$  mesons as well as of the charmonium states in hot nuclear medium can modify the decay of the charmonium states ( $\Psi', \chi_c, J/\Psi$ ) to  $D\bar{D}$  pairs in the hot dense hadronic matter. The small attractive potentials observed for the  $\bar{D}$  mesons may lead to formation of the  $\bar{D}$  mesic nuclei.

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## I. INTRODUCTION

The study of in-medium properties of hadrons is important for the understanding of strong interaction physics. The study of in-medium hadron properties has relevance in heavy-ion collision experiments as well as in nuclear astrophysics. There have been also extensive experimental efforts for the study of in-medium hadron properties by nuclear collision experiments. In these heavy-ion collision experiments, hot and dense matter is produced. By studying the experimental observables one can infer about how the hadron properties are modified in the medium. For example, the observed enhanced dilepton spectra [1–3] could be a signature of medium modifications of the vector mesons [4–8]. Similarly the properties of the kaons and antikaons have been studied experimentally by KaoS collaboration and the production of kaons and antikaons in the heavy-ion collisions and their collective flow are directly related to the medium modifications of their spectral functions [5, 9–15]. The study of  $D$  and  $\bar{D}$  mesons properties will be of direct relevance for the upcoming experiment at FAIR, GSI, where one expects to produce matter at high densities and moderate temperatures [16]. At such high densities, the properties of the  $D$  and  $\bar{D}$  mesons produced in these experiments are expected to be modified which should reflect in experimental observables like their production and propagation in the hot and dense medium. The reason for an expected appreciable modifications of the  $D$  and  $\bar{D}$  mesons is that  $D$  and  $\bar{D}$  mesons contain a light quark (u,d) or light antiquark. This light quark or antiquark interacts with the nuclear medium and leads to the modifications of  $D$  and  $\bar{D}$  properties. The experimental signature for this can be their production ratio and also in-medium  $J/\psi$  suppression [17–19]. In heavy-ion collision experiments of much higher collision energies, for example in RHIC or LHC, it is suggested that the  $J/\psi$  suppression is because of the formation of quark-gluon plasma (QGP) [20, 21]. However, in Ref. [22–24] it is observed that the effect of hadron absorption of  $J/\psi$  is not negligible. In Ref.[25], it was reported that the charmonium suppression observed in Pb + Pb collisions of NA50 experiment cannot be simply explained by nucleon absorption, but needs some additional density dependent suppression mechanism. It was suggested in these studies that the comover scattering [25–27] can explain the additional suppression of charmonium. An important difference between  $J/\psi$  suppression pattern in comovers interaction model and in a deconfining scenario is that, in the former case, the

anomalous suppression sets in smoothly from peripheral to central collisions rather than in a sudden way when the deconfining threshold is reached [26]. The  $J/\psi$  suppression in nuclear collisions at SPS energies has been studied in covariant transport approach HSD in Ref.[27]. The calculations show that the absorption of  $J/\psi$ 's by both nucleons and produced mesons can explain reasonably not only the total  $J/\psi$  cross-section but also the transverse energy dependence of  $J/\psi$  suppression measured in both proton-nucleus and nucleus collisions. In Ref.[28], the cross section of  $J/\psi$  dissociation by gluons is used to calculate the  $J/\psi$  suppression in an equilibrating parton gas produced in high energy nuclear collisions. The large average momentum in the hot gluon gas enables gluons to break up the  $J/\psi$ , while hadron matter at reasonable temperature does not provide sufficiently hard gluons.

Due to the reduction in the masses of  $D$  and  $\bar{D}$  mesons in the medium it is a possibility that excited charmonium states can decay to  $D\bar{D}$  pairs [29] instead of decaying to lowest charmonium state  $J/\psi$ . Actually higher charmonium states are considered as major source of  $J/\psi$  [30]. Even at certain higher densities, it can become a possibility that the  $J/\psi$  itself will decay to  $D\bar{D}$  pairs. So this can be an explanation of the observed  $J/\psi$  suppression by NA50 collaboration at 158 GeV/nucleon in the Pb-Pb collisions [20]. The excited states of charmonium also undergo mass drop in the nuclear medium [31]. The modifications of the in-medium masses of  $D$  mesons is large then the  $J/\psi$  mass modification [32, 33]. This is because the charmonium states are made up of a heavy charm quark and a charm antiquark. Within QCD sum rules, it is suggested that these heavy charmonium states interact with the nuclear medium through the gluon condensates. This is contrary to the interaction of the light vector mesons ( $\rho, \omega, \phi$ ), which interact with the nuclear medium through the quark condensates. This is because all the heavy quark condensates can be related to the gluon condensates via heavy-quark expansion [34]. Also in the nuclear medium there are no valence charm quark to leading order in density and any interaction with the medium is gluonic. The QCD sum rule approach [35] and leading order perturbative calculations [36] to study the medium modifications of charmonium, show that the mass of  $J/\psi$  is reduced slightly in the nuclear medium. In [31], the mass modification of charmonium has been studied using leading order QCD formula and the linear density approximation for the gluon condensate in the nuclear medium. This shows a small drop for the  $J/\psi$  mass at the nuclear matter density,

but there is seen to be significant shift in the masses of the excited states of charmonium ( $\psi(3686)$  and  $\psi(3770)$ ).

The in-medium modifications of  $D$  and  $\bar{D}$  mesons have been studied using various approaches. For example, in the QCD sum rule approach, it is suggested that the light quark or antiquark of  $D(\bar{D})$  mesons interacts with the light quark condensate leading to the medium modification of the  $D(\bar{D})$  meson masses [37, 38]. The quark meson coupling (QMC) model has also been used to study the D-meson properties [39]. In the QMC model, the light quarks (u,d) and light antiquarks ( $\bar{u},\bar{d}$ ) confined in the nucleons and mesons interact via exchange of a scalar-isoscalar  $\sigma$  meson as well a vector  $\omega$  meson. The nucleon has a large reduction of mass in the dense medium arising due to the interaction of the light quarks (u,d) with the  $\sigma$  field. The drop in the mass of  $D$  mesons observed in the QMC model turns out to be similar to those calculated within the QCD sum rule approach.

In the present investigation, we study the properties of the  $D$  and  $\bar{D}$  mesons in the isospin-asymmetric hot nuclear matter. These modifications arise due to their interactions with the nucleons, the non-strange scalar isoscalar meson  $\sigma$  and the scalar isovector meson  $\delta$ . We also study the medium modification of the masses of  $J/\psi$  and excited charmonium states  $\psi(3686)$  and  $\psi(3770)$  in the nuclear medium due to the interaction with the gluon condensates using the leading order QCD formula. The gluon condensate in the nuclear medium is calculated from the medium modification of a scalar dilaton field introduced within a chiral SU(3) model [40] through a scale symmetry breaking term in the Lagrangian density leading to the QCD trace anomaly. In the chiral SU(3) model, the gluon condensate is related to the fourth power of the dilaton field  $\chi$  and the changes in the dilaton field with the density are seen to be small. We study the isospin dependence of the in-medium masses of charmonium obtained from the dilaton field,  $\chi$  calculated for the asymmetric nuclear matter at finite temperatures. The medium modifications of the light hadrons (nucleons and scalar mesons) are described by using a chiral  $SU(3)$  model [40]. The model has been used to study finite nuclei, the nuclear matter properties, the in-medium properties of the vector mesons [41, 42] as well as to investigate the optical potentials of kaons and antikaons in nuclear matter [43, 44] and in hyperonic matter in [45]. For the study of the properties  $D$  mesons in isospin-asymmetric medium at finite temperatures, the chiral SU(3) model is

generalized to  $SU(4)$  flavor symmetry to obtain the interactions of  $D$  and  $\bar{D}$  mesons with the light hadrons. Since the chiral symmetry is explicitly broken for the  $SU(4)$  case due to the large charm quark mass, we use the  $SU(4)$  symmetry here only to obtain the interactions of the  $D$  and  $\bar{D}$  mesons with the light hadron sector, but use the observed values of the heavy hadron masses and empirical values of the decay constants. This has been in line with the philosophy followed in Ref. [46] where charmonium absorption in nuclear matter was studied using the  $SU(4)$  model to obtain the relevant interactions. However, the values of the heavy hadron masses and the coupling constants in Ref. [46], were taken as the empirical values or as calculated from other theoretical models. The coupling constants were derived by using the relations from  $SU(4)$  symmetry, if neither the empirical values nor values calculated from other theoretical models were available [46]. The  $D$  meson properties in symmetric hot nuclear matter using  $SU(4)$  model have been studied in ref. [47] and for the case of asymmetric nuclear matter at zero temperature in [48]. In a coupled channel approach for the study of  $D$  mesons, using a separable potential, it was shown that the resonance  $\Lambda_c(2593)$  is generated dynamically in the  $I=0$  channel [49] analogous to  $\Lambda(1405)$  in the coupled channel approach for the  $\bar{K}N$  interaction [50]. The approach has been generalized to study the spectral density of the  $D$ -mesons at finite temperatures and densities [51], taking into account the modifications of the nucleons in the medium. The results of this investigation seem to indicate a dominant increase in the width of the  $D$ -meson whereas there is only a very small change in the  $D$ -meson mass in the medium [51]. However, these calculations [49, 51], assume the interaction to be  $SU(3)$  symmetric in  $u, d, c$  quarks and ignore channels with charmed hadrons with strangeness. A coupled channel approach for the study of  $D$ -mesons has been developed based on  $SU(4)$  symmetry [52] to construct the effective interaction between pseudoscalar mesons in a 16-plet with baryons in 20-plet representation through exchange of vector mesons and with KSFR condition [53]. This model [52] has been modified in aspects like regularization method and has been used to study  $DN$  interactions in Ref. [54]. This reproduces the resonance  $\Lambda_c(2593)$  in the  $I=0$  channel and in addition generates another resonance in the  $I=1$  channel at around 2770 MeV. These calculations have been generalized to finite temperatures [55] accounting for the in-medium modifications of the nucleons in a Walecka type  $\sigma - \omega$  model, to study the  $D$  and  $\bar{D}$  properties [56] in the hot

and dense hadronic matter. At the nuclear matter density and for zero temperature, these resonances ( $\Lambda_c(2593)$  and  $\Sigma_c(2770)$ ) are generated 45 MeV and 40 MeV below their free space positions. However at finite temperature, e.g., at  $T = 100$  MeV resonance positions shift to 2579 MeV and 2767 MeV for  $\Lambda_c$  ( $I = 0$ ) and  $\Sigma_c$  ( $I = 1$ ) respectively. Thus at finite temperature resonances are seen to move closer to their free space values. This is because of the reduction of pauli blocking factor arising due to the fact that fermi surface is smeared out with temperature. For  $\bar{D}$  mesons in coupled channel approach a small repulsive mass shift is obtained. This will rule out of any possibility of charmed mesic nuclei [55] suggested in the QMC model [39]. But as we shall see in our investigation, we obtain a small attractive mass shift for  $\bar{D}$  mesons which can give rise to the possibility of the formation of charmed mesic nuclei. The study of  $D$  meson self-energy in the nuclear matter is also helpful in understanding the properties of the charm and the hidden charm resonances in the nuclear matter [57]. In coupled channel approach the charmed resonance  $D_{s0}(2317)$  mainly couples to  $DK$  system, while the  $D_0(2400)$  couples to  $D\pi$  and  $D_s\bar{K}$ . The hidden charm resonance couples mostly to  $D\bar{D}$ . Therefore any modification of  $D$  meson properties in the nuclear medium will affect the properties of these resonances. In Ref.[58, 59], the  $\bar{D}N$  interactions at low energies have been studied using a meson exchange model ( $\omega$  and  $\rho$ ) in close analogy with the meson exchange model for the  $KN$  interactions [60], supplemented with a short-distance contribution from one-gluon exchange. The scattering lengths for the  $I=0$  and  $I=1$  channels for the  $\bar{D}N$  interactions arising from the one gluon exchange are seen to be very close to the values in Ref. [56]. Generalizing the  $SU(4)$  models with vector meson exchange potentials to  $SU(8)$  spin-flavor symmetry, that treats the heavy pseudoscalar and vector mesons on equal footing-as required by heavy-quark symmetry, in Ref. [61], the charmed baryon resonances that are generated dynamically have been studied within a unitary meson-baryon coupled-channel model. Some of the resonances in this model are identified with the recently observed baryon resonances. In the present investigation, the  $D(\bar{D})$  energies are modified due to a vectorial Weinberg-Tomozawa, scalar exchange terms ( $\sigma$ ,  $\delta$ ) as well as range terms [44, 45]. The isospin asymmetric effects among  $D^0$  and  $D^+$  in the doublet,  $D \equiv (D^0, D^+)$  as well as between  $\bar{D}^0$  and  $D^-$  in the doublet,  $\bar{D} \equiv (\bar{D}^0, D^-)$  arise due to the scalar-isovector  $\delta$  meson, due to asymmetric contributions in the Weinberg-Tomozawa term,

as well as in the range term [44].

We organize the paper as follows. In section II, we give a brief introduction to the effective chiral  $SU(3)$  model used to study the isospin asymmetric nuclear matter at finite temperatures, and its extension to the  $SU(4)$  model to derive the interactions of the charmed mesons with the light hadrons. In section III, we present the dispersion relations for the  $D$  and  $\bar{D}$  mesons to be solved to calculate their optical potentials in the hot and dense hadronic matter. In section IV, we show how the in-medium masses of the charmonium states  $J/\psi$ ,  $\psi(3686)$  and  $\psi(3770)$  in the present investigation, arise from the medium modification of the scalar dilaton field, introduced within the chiral model to incorporate broken scale invariance leading to QCD trace anomaly. Section V contains the results and discussions and finally, in section VI, we summarize the results of present investigation and discuss possible outlook.

## II. THE HADRONIC CHIRAL $SU(3) \times SU(3)$ MODEL

We use a chiral  $SU(3)$  model for the study of the light hadrons in the present investigation [40]. The model is based on nonlinear realization of chiral symmetry [62–64] and broken scale invariance [40–42]. The effective hadronic chiral Lagrangian contains the following terms

$$\mathcal{L} = \mathcal{L}_{kin} + \sum_{W=X,Y,V,A,u} \mathcal{L}_{BW} + \mathcal{L}_{vec} + \mathcal{L}_0 + \mathcal{L}_{SB} \quad (1)$$

In Eq.(1),  $\mathcal{L}_{kin}$  is the kinetic energy term,  $\mathcal{L}_{BW}$  is the baryon-meson interaction term in which the baryons-spin-0 meson interaction term generates the baryon masses.  $\mathcal{L}_{vec}$  describes the dynamical mass generation of the vector mesons via couplings to the scalar mesons and contain additionally quartic self-interactions of the vector fields.  $\mathcal{L}_0$  contains the meson-meson interaction terms inducing the spontaneous breaking of chiral symmetry as well as a scale invariance breaking logarithmic potential.  $\mathcal{L}_{SB}$  describes the explicit chiral symmetry breaking.

To study the hadron properties at finite temperature and densities in the present investigation, we use the mean field approximation, where all the meson fields are treated as classical fields. In this approximation, only the scalar and the vector fields contribute to the baryon-meson interaction,  $\mathcal{L}_{BW}$  since for all the other mesons, the expectation values are



zero. The interactions of the scalar mesons and vector mesons with the baryons are given as

$$\mathcal{L}_{Bscal} + \mathcal{L}_{Bvec} = - \sum_i \bar{\psi}_i [m_i^* + g_{\omega i} \gamma_0 \omega + g_{\rho i} \gamma_0 \rho + g_{\phi i} \gamma_0 \phi] \psi_i. \quad (2)$$

The interaction of the vector mesons, of the scalar fields and the interaction corresponding to the explicitly symmetry breaking in the mean field approximation are given as

$$\begin{aligned} \mathcal{L}_{vec} = & \frac{1}{2} \left( m_\omega^2 \omega^2 + m_\rho^2 \rho^2 + m_\phi^2 \phi^2 \right) \frac{\chi^2}{\chi_0^2} \\ & + g_4 (\omega^4 + 6\omega^2 \rho^2 + \rho^4 + 2\phi^4), \end{aligned} \quad (3)$$

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{2} k_0 \chi^2 (\sigma^2 + \zeta^2 + \delta^2) + k_1 (\sigma^2 + \zeta^2 + \delta^2)^2 \\ & + k_2 \left( \frac{\sigma^4}{2} + \frac{\delta^4}{2} + 3\sigma^2 \delta^2 + \zeta^4 \right) + k_3 \chi (\sigma^2 - \delta^2) \zeta \\ & - k_4 \chi^4 - \frac{1}{4} \chi^4 \ln \frac{\chi^4}{\chi_0^4} + \frac{d}{3} \chi^4 \ln \left( \left( \frac{(\sigma^2 - \delta^2) \zeta}{\sigma_0^2 \zeta_0} \right) \left( \frac{\chi}{\chi_0} \right)^3 \right), \end{aligned} \quad (4)$$

and

$$\mathcal{L}_{SB} = - \left( \frac{\chi}{\chi_0} \right)^2 \left[ m_\pi^2 f_\pi \sigma + \left( \sqrt{2} m_k^2 f_k - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right) \zeta \right]. \quad (5)$$

In (2),  $m_i^*$  is the effective mass of the baryon of species  $i$ , given as

$$m_i^* = -(g_{\sigma i} \sigma + g_{\zeta i} \zeta + g_{\delta i} \delta) \quad (6)$$

The baryon-scalar meson interactions, as can be seen from equation (6), generate the baryon masses through the coupling of baryons to the non-strange  $\sigma$ , strange  $\zeta$  scalar mesons and also to scalar-isovector meson  $\delta$ . In analogy to the baryon-scalar meson coupling there exist two independent baryon-vector meson interaction terms corresponding to the F-type (antisymmetric) and D-type (symmetric) couplings. Here antisymmetric coupling is used because the universality principle [65] and vector meson dominance model suggest small symmetric coupling. Additionally, we choose the parameters [40, 44] so as to decouple the strange vector field  $\phi_\mu \sim \bar{s} \gamma_\mu s$  from the nucleon, corresponding to an ideal mixing between  $\omega$  and  $\phi$  mesons. A small deviation of the mixing angle from ideal mixing [66–68] has not been taken into account in the present investigation.

The concept of broken scale invariance leading to the trace anomaly in (massless) QCD,  $\theta_\mu^\mu = \frac{\beta_{QCD}}{2g} G_{\mu\nu}^a G^{\mu\nu a}$ , where  $G_{\mu\nu}^a$  is the gluon field strength tensor of QCD, is simulated in the effective Lagrangian at tree level [69] through the introduction of the scale breaking terms

$$\mathcal{L}_{scalebreaking} = -\frac{1}{4}\chi^4 \ln\left(\frac{\chi^4}{\chi_0^4}\right) + \frac{d}{3}\chi^4 \ln\left(\left(\frac{I_3}{\det\langle X \rangle_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right), \quad (7)$$

where  $I_3 = \det\langle X \rangle$ , with  $X$  as the multiplet for the scalar mesons. These scale breaking terms, in the mean field approximation, are given by the last two terms of the Lagrangian density,  $\mathcal{L}_0$  given by equation (4). The effect of these logarithmic terms is to break the scale invariance, which leads to the trace of the energy momentum tensor as [70]

$$\theta_\mu^\mu = \chi \frac{\partial \mathcal{L}}{\partial \chi} - 4\mathcal{L} = -(1-d)\chi^4. \quad (8)$$

Hence the scalar gluon condensate of QCD ( $\frac{\alpha_s}{\pi} \langle G_{\mu\nu}^a G^{\mu\nu a} \rangle$ ) is simulated by a scalar dilaton field in the present hadronic model.

The coupled equations of motion for the non-strange scalar field  $\sigma$ , strange scalar field  $\zeta$ , scalar-isovector field  $\delta$  and dilaton field  $\chi$ , are derived from the Lagrangian density and are given as

$$\begin{aligned} & k_0\chi^2\sigma - 4k_1(\sigma^2 + \zeta^2 + \delta^2)\sigma - 2k_2(\sigma^3 + 3\sigma\delta^2) - 2k_3\chi\sigma\zeta \\ & - \frac{d}{3}\chi^4\left(\frac{2\sigma}{\sigma^2 - \delta^2}\right) + \left(\frac{\chi}{\chi_0}\right)^2 m_\pi^2 f_\pi - \sum g_{\sigma i} \rho_i^s = 0 \end{aligned} \quad (9)$$

$$\begin{aligned} & k_0\chi^2\zeta - 4k_1(\sigma^2 + \zeta^2 + \delta^2)\zeta - 4k_2\zeta^3 - k_3\chi(\sigma^2 - \delta^2) \\ & - \frac{d}{3}\frac{\chi^4}{\zeta} + \left(\frac{\chi}{\chi_0}\right)^2 \left[ \sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi \right] - \sum g_{\zeta i} \rho_i^s = 0 \end{aligned} \quad (10)$$

$$\begin{aligned} & k_0\chi^2\delta - 4k_1(\sigma^2 + \zeta^2 + \delta^2)\delta - 2k_2(\delta^3 + 3\sigma^2\delta) + k_3\chi\delta\zeta \\ & + \frac{2}{3}d\chi^4\left(\frac{\delta}{\sigma^2 - \delta^2}\right) - \sum g_{\delta i} \rho_i^s = 0 \end{aligned} \quad (11)$$

$$\begin{aligned} & k_0\chi(\sigma^2 + \zeta^2 + \delta^2) - k_3(\sigma^2 - \delta^2)\zeta + \chi^3 \left[ 1 + \ln\left(\frac{\chi^4}{\chi_0^4}\right) \right] + (4k_4 - d)\chi^3 \\ & - \frac{4}{3}d\chi^3 \ln\left(\left(\frac{(\sigma^2 - \delta^2)\zeta}{\sigma_0^2\zeta_0}\right)\left(\frac{\chi}{\chi_0}\right)^3\right) + \frac{2\chi}{\chi_0^2} \left[ m_\pi^2 f_\pi \sigma + \left( \sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi \right) \zeta \right] = 0 \end{aligned} \quad (12)$$

In the above,  $\rho_i^s$  are the scalar densities for the baryons, given as

$$\rho_i^s = \gamma_i \int \frac{d^3k}{(2\pi)^3} \frac{m_i^*}{E_i^*(k)} \left( \frac{1}{e^{(E_i^*(k)-\mu_i^*)/T} + 1} + \frac{1}{e^{(E_i^*(k)+\mu_i^*)/T} + 1} \right) \quad (13)$$

where,  $E_i^*(k) = (k^2 + m_i^{*2})^{1/2}$ , and,  $\mu_i^* = \mu_i - g_{\omega i}\omega - g_{\rho i}\rho - g_{\phi i}\phi$ , are the single particle energy and the effective chemical potential for the baryon of species  $i$ , and,  $\gamma_i=2$  is the spin degeneracy factor [44].

The above coupled equations of motion are solved to obtain the density and temperature dependent values of the scalar fields ( $\sigma$ ,  $\zeta$  and  $\delta$ ) and the dilaton field,  $\chi$ , in the isospin asymmetric hot nuclear medium. As has already been mentioned, the value of the  $\chi$  is related to the scalar gluon condensate in the hot hadronic medium, and is used to compute the in-medium masses of charmonium states in the present investigation. The isospin asymmetry in the medium is introduced through the scalar-isovector field  $\delta$  and therefore the dilaton field obtained after solving the above equations is also dependent on the isospin asymmetry parameter,  $\eta$  defined as  $\eta = (\rho_n - \rho_p)/(2\rho_B)$ , where  $\rho_n$  and  $\rho_p$  are the number densities of the neutron and the proton and  $\rho_B$  is the baryon density. In the present investigation, we study the effect of isospin asymmetry of the medium on the masses of the charmonium states  $J/\psi$ ,  $\psi(3686)$  and  $\psi(3770)$ .

The comparison of the trace of the energy momentum tensor arising from the trace anomaly of QCD with that of the present chiral model gives the relation of the dilaton field to the scalar gluon condensate. We have, in the limit of massless quarks [71],

$$\theta_\mu^\mu = \langle \frac{\beta_{QCD}}{2g} G_{\mu\nu}^a G^{\mu\nu a} \rangle \equiv -(1-d)\chi^4 \quad (14)$$

The parameter  $d$  originates from the second logarithmic term of equation (7). To get an insight into the value of the parameter  $d$ , we recall that the QCD  $\beta$  function at one loop level, for  $N_c$  colors and  $N_f$  flavors is given by

$$\beta_{QCD}(g) = -\frac{11N_c g^3}{48\pi^2} \left( 1 - \frac{2N_f}{11N_c} \right) + O(g^5) \quad (15)$$

In the above equation, the first term in the parentheses arises from the (antiscreening) self-interaction of the gluons and the second term, proportional to  $N_f$ , arises from the (screening) contribution of quark pairs. Equations (14) and (15) suggest the value of  $d$  to be  $6/33$  for three flavors and three colors, and for the case of three colors and two flavors, the value

of  $d$  turns out to be  $4/33$ , to be consistent with the one loop estimate of QCD  $\beta$  function. These values give the order of magnitude about which the parameter  $d$  can be taken [70], since one cannot rely on the one-loop estimate for  $\beta_{\text{QCD}}(g)$ . In the present investigation of the in-medium properties of the charmonium states due to the medium modification of the dilaton field within chiral  $SU(3)$  model, we use the value of  $d=0.064$  [48]. This parameter, along with the other parameters corresponding to the scalar Lagrangian density,  $\mathcal{L}_0$  given by (4), are fitted so as to ensure extrema in the vacuum for the  $\sigma$ ,  $\zeta$  and  $\chi$  field equations, to reproduce the vacuum masses of the  $\eta$  and  $\eta'$  mesons, the mass of the  $\sigma$  meson around 500 MeV, and pressure,  $p(\rho_0)=0$ , with  $\rho_0$  as the nuclear matter saturation density [40, 48].

The trace of the energy-momentum tensor in QCD, using the one loop beta function given by equation (15), for  $N_c=3$  and  $N_f=3$ , is given as,

$$\theta_\mu^\mu = -\frac{9}{8} \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{\mu\nu a} \quad (16)$$

Using equations (14) and (16), we can write

$$\left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{\mu\nu a} \right\rangle = \frac{8}{9} (1-d) \chi^4 \quad (17)$$

We thus see from the equation (17) that the scalar gluon condensate  $\left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{\mu\nu a} \right\rangle$  is proportional to the fourth power of the dilaton field,  $\chi$ , in the chiral  $SU(3)$  model. As mentioned earlier, the in-medium masses of charmonium states are modified due to the gluon condensates. Therefore, we need to know the change in the gluon condensate with density and temperature of the asymmetric nuclear medium, which is calculated from the modification of the  $\chi$  field, by using equation (17).

### III. $D$ AND $\bar{D}$ MESONS IN HOT ASYMMETRIC NUCLEAR MATTER

In this section we study the  $D$  and  $\bar{D}$  mesons properties in isospin-asymmetric nuclear matter at finite temperatures. The medium modifications of the  $D$  and  $\bar{D}$  mesons arise due to their interactions with the nucleons and the scalar mesons and the interaction Lagrangian density is given as [48]

$$\mathcal{L}_{\mathcal{DN}} = -\frac{i}{8f_D^2} \left[ 3(\bar{p}\gamma^\mu p + \bar{n}\gamma^\mu n) (D^0(\partial_\mu \bar{D}^0) - (\partial_\mu D^0)\bar{D}^0) + (D^+(\partial_\mu D^-) - (\partial_\mu D^+)D^-) \right]$$

$$\begin{aligned}
& + \left( \bar{p}\gamma^\mu p - \bar{n}\gamma^\mu n \right) \left( D^0(\partial_\mu \bar{D}^0) - (\partial_\mu D^0)\bar{D}^0 \right) - \left( D^+(\partial_\mu D^-) - (\partial_\mu D^+)D^- \right) \\
& + \frac{m_D^2}{2f_D} \left[ (\sigma + \sqrt{2}\zeta_c)(\bar{D}^0 D^0 + (D^- D^+)) + \delta(\bar{D}^0 D^0) - (D^- D^+) \right] \\
& - \frac{1}{f_D} \left[ (\sigma + \sqrt{2}\zeta_c) \left( (\partial_\mu \bar{D}^0)(\partial^\mu D^0) + (\partial_\mu D^-)(\partial^\mu D^+) \right) \right. \\
& + \left. \delta \left( (\partial_\mu \bar{D}^0)(\partial^\mu D^0) - (\partial_\mu D^-)(\partial^\mu D^+) \right) \right] \\
& + \frac{d_1}{2f_D^2} (\bar{p}p + \bar{n}n) \left( (\partial_\mu D^-)(\partial^\mu D^+) + (\partial_\mu \bar{D}^0)(\partial^\mu D^0) \right) \\
& + \frac{d_2}{4f_D^2} \left[ (\bar{p}p + \bar{n}n) \left( (\partial_\mu \bar{D}^0)(\partial^\mu D^0) + (\partial_\mu D^-)(\partial^\mu D^+) \right) \right. \\
& + \left. (\bar{p}p - \bar{n}n) \left( (\partial_\mu \bar{D}^0)(\partial^\mu D^0) - (\partial_\mu D^-)(\partial^\mu D^+) \right) \right]
\end{aligned} \tag{18}$$

In Eq.(18), the first term is the vectorial Weinberg Tomozawa interaction term, obtained from the kinetic term of Eq.(1). The second term is obtained from the explicit symmetry breaking term and leads to the attractive interactions for both the  $D$  and  $\bar{D}$  mesons in the medium. The next three terms of above Lagrangian density ( $\sim (\partial_\mu \bar{D})(\partial^\mu D)$ ) are known as the range terms. The first range term (with coefficient  $(-\frac{1}{f_D})$ ) is obtained from the kinetic energy term of the pseudoscalar mesons. The second and third range terms  $d_1$  and  $d_2$  are written for the  $DN$  interactions in analogy with those written for  $KN$  interactions in [45]. It might be noted here that the interaction of the pseudoscalar mesons with the vector mesons, in addition to the pseudoscalar meson-nucleon vectorial interaction, leads to a double counting in the linear realization of chiral effective theories. Further, in the non-linear realization, such an interaction does not arise in the leading or subleading order, but only as a higher order contribution [72]. Hence the vector meson-pseudoscalar interactions will not be taken into account in the present investigation.

The dispersion relations for the  $D$  and  $\bar{D}$  mesons are obtained by the Fourier transformations of equations of motion. These are given as

$$-\omega^2 + \vec{k}^2 + m_D^2 - \Pi(\omega, |\vec{k}|) = 0 \tag{19}$$

where,  $m_D$  is the vacuum mass of the  $D(\bar{D})$  meson taken as 1869 MeV and 1864.5 MeV for the  $D$  and  $\bar{D}$  mesons respectively.  $\Pi(\omega, |\vec{k}|)$  denotes the self-energy of the  $D(\bar{D})$  mesons in the medium.

The self-energy  $\Pi(\omega, |\vec{k}|)$  for the  $D$  meson doublet ( $D^0, D^+$ ) arising from the interaction

of Eq.(18) is given as

$$\begin{aligned}
\Pi(\omega, |\vec{k}|) = & \frac{1}{4f_D^2} [3(\rho_p + \rho_n) \pm (\rho_p - \rho_n)] \omega \\
& + \frac{m_D^2}{2f_D} (\sigma' + \sqrt{2}\zeta_c' \pm \delta') \\
& + \left[ -\frac{1}{f_D} (\sigma' + \sqrt{2}\zeta_c' \pm \delta') + \frac{d_1}{2f_D^2} (\rho_p^s + \rho_n^s) \right. \\
& \left. + \frac{d_2}{4f_D^2} ((\rho_p^s + \rho_n^s) \pm (\rho_p^s - \rho_n^s)) \right] (\omega^2 - \vec{k}^2), \tag{20}
\end{aligned}$$

where the  $\pm$  signs refer to the  $D^0$  and  $D^+$  mesons, respectively, and  $\sigma' (\sigma - \sigma_0)$ ,  $\zeta_c' (\zeta_c - \zeta_{c0})$ , and  $\delta' (= \delta - \delta_0)$  are the fluctuations of the scalar isoscalar fields  $\sigma$  and  $\zeta$  and the scalar-isoscalar field  $\delta$  from their vacuum expectation values. The vacuum expectation value of  $\delta$  is zero ( $\delta_0 = 0$ ), since a nonzero value for it will break the isospin-symmetry of the vacuum. (We neglect here the small isospin breaking effect arising from the mass and charge difference of the up and down quarks.) We might note here that the interaction of the scalar quark condensate  $\zeta_c$  (being made up of heavy charmed quarks and antiquarks) leads to very small modifications of the masses [73]. So we will not consider the medium fluctuations of  $\zeta_c$ . In the present investigation, we take the value of the D meson decay constant,  $f_D$  as 135 MeV [74]. Within the present model, the medium modification to the  $D(\bar{D})$  mesons due to the scalar interaction depends only on the fluctuations of the scalar  $\sigma$  and  $\delta$  fields in the asymmetric hot nuclear medium which are determined by solving the coupled equations (9), (10), (11) and (12). As the scalar term of (20) does not contain any free parameters, with the assumption that the fluctuation of the charm condensate has a negligible effect on the masses of the  $D(\bar{D})$  mesons, there are no uncertainties in the mass shift due to the scalar interaction in the present investigation, once the value for  $f_D$  is chosen. In Eq. (20),  $\rho_p$  and  $\rho_n$  are the number densities of protons and neutrons given by

$$\rho_i = \gamma_i \int \frac{d^3k}{(2\pi)^3} \left( \frac{1}{e^{(E_i^*(k) - \mu_i^*)/T} + 1} - \frac{1}{e^{(E_i^*(k) + \mu_i^*)/T} + 1} \right), \tag{21}$$

for  $i=p$  and  $n$ , and  $\rho_p^s$  and  $\rho_n^s$  are their scalar densities, as given by equation (13).

Similarly, for the  $\bar{D}$  meson doublet ( $\bar{D}^0, D^-$ ), the self-energy is calculated as

$$\Pi(\omega, |\vec{k}|) = -\frac{1}{4f_D^2} [3(\rho_p + \rho_n) \pm (\rho_p - \rho_n)] \omega$$

$$\begin{aligned}
& + \frac{m_D^2}{2f_D}(\sigma' + \sqrt{2}\zeta_c' \pm \delta') \\
& + \left[ -\frac{1}{f_D}(\sigma' + \sqrt{2}\zeta_c' \pm \delta') + \frac{d_1}{2f_D^2}(\rho_s^p + \rho_s^n) \right. \\
& \left. + \frac{d_2}{4f_D^2}((\rho_p^s + \rho_n^s) \pm (\rho_p^s - \rho_n^s)) \right](\omega^2 - \vec{k}^2), \tag{22}
\end{aligned}$$

where the  $\pm$  signs refer to the  $\bar{D}^0$  and  $D^-$  mesons, respectively. The optical potentials of the  $D$  and  $\bar{D}$  mesons are obtained using the expression

$$U(\omega, k) = \omega(k) - \sqrt{k^2 + m_D^2} \tag{23}$$

where  $m_D$  is the vacuum mass for the  $D(\bar{D})$  meson and  $\omega(k)$  is the momentum-dependent energy of the  $D(\bar{D})$  meson.

#### IV. CHARMONIUM MASSES IN HOT ASYMMETRIC NUCLEAR MATTER

In this section, we investigate the masses of charmonium states  $J/\psi$ ,  $\psi(3686)$  and  $\psi(3770)$ , in isospin asymmetric hot nuclear matter. From the QCD sum rule calculations, the mass shift of the charmonium states in the medium is due to the gluon condensates [31, 37]. For heavy quark systems, there are two independent lowest dimension operators: the scalar gluon condensate ( $\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{\mu\nu a} \rangle$ ) and the condensate of the twist 2 gluon operator ( $\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{\mu\alpha\alpha} \rangle$ ). These operators can be rewritten in terms of the color electric and color magnetic fields,  $\langle \frac{\alpha_s}{\pi} \vec{E}^2 \rangle$  and  $\langle \frac{\alpha_s}{\pi} \vec{B}^2 \rangle$ . Additionally, since the Wilson coefficients for the operator  $\langle \frac{\alpha_s}{\pi} \vec{B}^2 \rangle$  vanish in the non-relativistic limit, the only contribution from the gluon condensates is proportional to  $\langle \frac{\alpha_s}{\pi} \vec{E}^2 \rangle$ , similar to the second order Stark effect. Hence, the mass shift of the charmonium states arises due to the change in the operator  $\langle \frac{\alpha_s}{\pi} \vec{E}^2 \rangle$  in the medium from its vacuum value [31]. In the leading order mass shift formula derived in the large charm mass limit [36], the shift in the mass of the charmonium state is given as [31]

$$\Delta m_\psi(\epsilon) = -\frac{1}{9} \int dk^2 \left| \frac{\partial \psi(k)}{\partial k} \right|^2 \frac{k}{k^2/m_c + \epsilon} \left( \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle - \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_0 \right). \tag{24}$$

In the above,  $m_c$  is the mass of the charm quark, taken as 1.95 GeV [31],  $m_\psi$  is the vacuum mass of the charmonium state and  $\epsilon = 2m_c - m_\psi$ .  $\psi(k)$  is the wave function of the charmonium state in the momentum space, normalized as  $\int \frac{d^3k}{2\pi^3} |\psi(k)|^2 = 1$  [75]. At finite

densities, in the linear density approximation, the change in the value of  $\langle \frac{\alpha_s}{\pi} \vec{E}^2 \rangle$ , from its vacuum value, is given as

$$\left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle - \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_0 = \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_N \frac{\rho_B}{2M_N}, \quad (25)$$

and the mass shift in the charmonium states reduces to [31]

$$\Delta m_\psi(\epsilon) = -\frac{1}{9} \int dk^2 \left| \frac{\partial \psi(k)}{\partial k} \right|^2 \frac{k}{k^2/m_c + \epsilon} \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_N \frac{\rho_B}{2M_N}. \quad (26)$$

In the above,  $\left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_N$  is the expectation value of  $\left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle$  with respect to the nucleon.

The expectation value of the scalar gluon condensate can be expressed in terms of the color electric field and the color magnetic field as [76]

$$\left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{\mu\nu a} \right\rangle = -2 \left\langle \frac{\alpha_s}{\pi} (E^2 - B^2) \right\rangle. \quad (27)$$

In the non-relativistic limit, as already mentioned, the contribution from the magnetic field vanishes and hence, we can write,

$$\left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle = -\frac{1}{2} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{\mu\nu a} \right\rangle \quad (28)$$

Using equations (17), (24) and (28), we obtain the expression for the mass shift in the charmonium in the hot and dense nuclear medium, which arises from the change in the dilaton field in the present investigation, as

$$\Delta m_\psi(\epsilon) = \frac{4}{81} (1-d) \int dk^2 \left| \frac{\partial \psi(k)}{\partial k} \right|^2 \frac{k}{k^2/m_c + \epsilon} (\chi^4 - \chi_0^4). \quad (29)$$

In the above,  $\chi$  and  $\chi_0$  are the values of the dilaton field in the nuclear medium and in the vacuum respectively.

In the present investigation, the wave functions for the charmonium states are taken to be Gaussian and are given as [33]

$$\psi_{N,l} = \text{Normalization} \times Y_l^m(\theta, \phi) (\beta^2 r^2)^{\frac{1}{2}l} \exp^{-\frac{1}{2}\beta^2 r^2} L_{N-1}^{l+\frac{1}{2}}(\beta^2 r^2) \quad (30)$$

where  $\beta^2 = M\omega/h$  characterizes the strength of the harmonic potential,  $M = m_c/2$  is the reduced mass of the charm quark and charm anti-quark system, and  $L_p^k(z)$  is the associated Laguerre Polynomial. As in Ref. [31], the oscillator constant  $\beta$  is determined from



the mean squared radii  $\langle r^2 \rangle$  as  $0.46^2 \text{ fm}^2$ ,  $0.96^2 \text{ fm}^2$  and  $1 \text{ fm}^2$  for the charmonium states  $J/\psi(3097)$ ,  $\psi(3686)$  and  $\psi(3770)$ , respectively. This gives the value for the parameter  $\beta$  as 0.51 GeV, 0.38 GeV and 0.37 GeV for  $J/\psi(3097)$ ,  $\psi(3686)$  and  $\psi(3770)$ , assuming that these charmonium states are in the 1S, 2S and 1D states respectively. Knowing the wave functions of the charmonium states and calculating the medium modification of the dilaton field in the hot nuclear matter, we obtain the mass shift of the charmonium states,  $J/\psi$ ,  $\psi(3686)$  and  $\psi(3770)$  respectively. In the next section we shall present the results of the present investigation of these in-medium charmonium masses in hot asymmetric nuclear matter.

## V. RESULTS AND DISCUSSIONS

In this section, we present the results of our investigation for the in-medium masses of  $D$  and  $\bar{D}$  mesons as well as of the charmonium states  $J/\psi(3097)$ ,  $\psi(3686)$  and  $\psi(3770)$ , in isospin asymmetric nuclear matter at finite temperatures. We have generalized the chiral  $SU(3)$  model to  $SU(4)$  to include the interactions of the charmed mesons. The values of the parameters used in the present investigation, are :  $k_0 = 2.54, k_1 = 1.35, k_2 = -4.78, k_3 = -2.77, k_4 = -0.22$  and  $d = 0.064$ , which are the parameters occurring in the scalar meson interactions defined in equation (4). The vacuum values of the scalar isoscalar fields,  $\sigma$  and  $\zeta$  and the dilaton field  $\chi$  are  $-93.3 \text{ MeV}$ ,  $-106.6 \text{ MeV}$  and  $409.8 \text{ MeV}$  respectively. The values,  $g_{\sigma N} = 10.6$  and  $g_{\zeta N} = -0.47$  are determined by fitting to vacuum baryon masses. The other parameters fitted to the asymmetric nuclear matter saturation properties in the mean-field approximation are:  $g_{\omega N} = 13.3, g_{\rho p} = 5.5, g_4 = 79.7, g_{\delta p} = 2.5, m_\zeta = 1024.5 \text{ MeV}, m_\sigma = 466.5 \text{ MeV}$  and  $m_\delta = 899.5 \text{ MeV}$ . The nuclear matter saturation density used in the present investigation is  $0.15 \text{ fm}^{-3}$ . The coefficients  $d_1$  and  $d_2$ , calculated from the empirical values of the  $KN$  scattering lengths for  $I = 0$  and  $I = 1$  channels, are  $2.56/m_K$  and  $0.73/m_K$ , respectively [45].

In isospin asymmetric nuclear medium, the properties of the  $D$  mesons ( $D^0, D^+$ ) and  $\bar{D}$  mesons ( $\bar{D}^0, D^-$ ), due to their interactions with the hot hadronic medium, undergo medium modifications. These modifications arise due to the interactions with the nucleons (through the Weinberg-Tomozawa vectorial interaction as well as through the range terms) and the scalar exchange terms. The modifications of the scalar mean fields modify the masses of the

nucleons in the hot and dense hadronic medium. Before going into the details of how the  $D$  and  $\bar{D}$  mesons properties are modified at finite temperatures in the dense nuclear medium, let us see how the scalar fields are modified at finite temperatures in the nuclear medium. In figures 1, 2, 3 and 4, we show the variation of scalar fields  $\sigma$ ,  $\zeta$  and scalar-isovector field  $\delta$  and the dilaton field  $\chi$ , with temperature, for both zero and finite densities, and for selected values of the isospin asymmetry parameter,  $\eta = 0, 0.1, 0.3$  and  $0.5$ . At zero baryon density, we observe that the magnitudes of the scalar fields  $\sigma$  and  $\zeta$  decrease with increase in temperature. However, the drop in their magnitudes with temperature is negligible upto a temperature of about 100 MeV (that is, they remain very close to their vacuum values). The changes in the magnitudes of the  $\sigma$  and  $\zeta$  fields are 2.5 MeV and 0.8 MeV respectively, when the temperature changes from 100 MeV to 150 MeV, above which the drop increases. These values change to about 10 MeV and 3 MeV for the  $\sigma$  and  $\zeta$  fields respectively, for a change in temperature of 100 MeV to 175 MeV. At zero baryon density, it is observed that the value of the dilaton field remains almost constant upto a temperature of about 130 MeV above which it is seen to drop with increase in temperature. However, the drop in the dilaton field is seen to be very small. The value of the dilaton field is seen to change from 409.8 MeV at  $T = 0$  to about 409.7 MeV and 409.3 MeV at  $T = 150$  MeV and  $T = 175$  MeV, respectively. The thermal distribution functions have an effect of increasing the scalar densities at zero baryon density, i.e.  $\mu_i^* = 0$ , as can be seen from the expression of the scalar densities, given by equation (13). This effect seems to be negligible upto a temperature of about 130 MeV. This leads to a decrease in the magnitudes of the scalar fields,  $\sigma$  and  $\zeta$ . This behaviour of the scalar fields is reflected in the value of  $\chi$ , which is solved from the coupled equations of motion of the scalar fields, given by equations (9), (10), (11) and (12), as a drop as we increase the temperature above a temperature of about 130 MeV. The scalar densities attaining nonzero values at high temperatures, even at zero baryon density, indicates the presence of baryon-antibaryon pairs in the thermal bath and has already been observed in literature [42, 77]. This leads to the baryon masses to be different from their vacuum masses above this temperature, arising from modifications of the scalar fields  $\sigma$  and  $\zeta$ .

For finite density situation, the behaviour of the scalar fields with temperature is seen to

be very different from the zero density case, as can be seen from the subplots (b), (c) and (d) of figures 1, 2 and 4 for  $\sigma$ ,  $\zeta$  and  $\chi$  fields respectively, where the fields are plotted as functions of temperature for densities  $\rho_0$ ,  $2\rho_0$  and  $4\rho_0$  respectively. At finite densities, one observes first a rise and then a decrease of the scalar fields  $\sigma$ ,  $\zeta$  and  $\chi$  with temperature. For example, at  $\rho_B = \rho_0$ , and for the value of the isospin asymmetry parameter,  $\eta = 0$  the scalar fields  $\sigma$ ,  $\zeta$  and  $\chi$  increase with temperature upto a temperature of about 145 MeV and above this temperature, they both start decreasing. At  $\eta = 0.5$  the value of temperature upto which these scalar fields increase becomes about 120 MeV. For  $\rho_B = 4\rho_0$  and  $\eta = 0$  the scalar fields  $\sigma$ ,  $\zeta$  and  $\chi$  increase upto a temperature of about 160 MeV. At  $\eta = 0.5$  this value of temperature is lowered to about 120 MeV for  $\sigma$ ,  $\zeta$  and  $\chi$  fields respectively. This observed rise in the magnitudes of  $\sigma$  and  $\zeta$  fields with temperature leads to an increase in the mass of nucleons with temperature at finite densities. The reason for the different behaviour of the scalar fields ( $\sigma$  and  $\zeta$ ) at zero and finite densities can be understood in the following manner [42]. As has already been mentioned, the thermal distribution functions in (13) have an effect of increasing the scalar densities at zero baryon density, i.e., for  $\mu_i^* = 0$ . However, at finite densities, i.e. for nonzero values of the effective chemical potential,  $\mu_i^*$ , for increasing temperature, there are contributions also from higher momenta, thereby, increasing the denominator of the integrand on the right hand side of equation (13). This leads to a decrease in the scalar density. The competing effects of the thermal distribution functions and the contributions from the higher moments states give rise to observed behaviour of the scalar density and hence of the  $\sigma$  and  $\zeta$  fields with temperature at finite baryon densities [42]. This kind of behaviour of the scalar  $\sigma$  field on temperature at finite densities has also been observed in the Walecka model by Li and Ko [8]. The behaviour of the scalar fields  $\sigma$  and  $\zeta$  with the temperature is reflected in the behaviour of  $\chi$  field, since it is solved from the coupled equations of the scalar fields. In figure 3, we observe that the value of the scalar isovector field,  $\delta$  is zero at zero baryon density, since there is no isospin asymmetry at zero density. At finite baryon densities the magnitude of the scalar isovector field,  $\delta$  decreases with increase in the temperature of the nuclear medium. However, for given temperature as we move to higher densities the magnitude of the  $\delta$  field is seen to increase which means that the medium is more asymmetric at higher densities. In the isospin asymmetric nuclear

medium, the scalar isovector  $\delta$  meson attains a nonzero expectation value. However, the magnitudes of the  $\delta$  field at a given baryon density and temperature of the medium, which are solved from the coupled equations, (9) to (12), for the scalar fields, turn out to be small ( $\leq 4 - 5 \text{ MeV}$ ), as can be seen from figure 3. This causes changes in the  $\sigma$  and  $\zeta$  fields, as can be seen from figures 1 and 2, also to be small in the isospin asymmetric nuclear matter, since these solved from the equations (9) and (10), along with the equations for  $\delta$  and  $\chi$ , given by (11) and (12).

In figures 5 and 6, we show the variation of the energy of  $D$  and  $\bar{D}$  mesons due to Weinberg-Tomozawa term, the scalar exchange term arising from the explicit symmetry breaking term and the range terms, as functions of temperature and for the densities  $\rho_B = 0, \rho_0$  and  $4\rho_0$ . These are plotted asymmetric nuclear matter with the value of the isospin asymmetry parameter as 0.5, and in the same figure, the results arising from these contributions for the isospin symmetric case ( $\eta=0$ ) are shown as the dotted lines. At zero density, the contributions from the Weinberg-Tomozawa term is zero, since the densities of the proton and neutron,  $\rho_p$  and  $\rho_n$  are zero at zero density. The scalar fields attaining values different from their vacuum values at zero density at values of temperature higher than about 100 MeV, as already observed in figures 1 and 2, leads to a decrease in the  $D^+$  and  $D^0$  mesons due to the scalar meson exchange term and an increase in the contribution due to the range term, as can be seen in the subplots (a) and (b) of figure 5. These two terms seem to almost cancel with each other, resulting in negligible modifications of  $D^+$  and  $D^0$  mesons. The drop in the masses of  $D^+$  and  $D^0$  are seen to be about 0.75 MeV and 0.75 MeV respectively at temperature  $T=150$  MeV and about 5.4 MeV and 5.35 MeV at a temperature of 175 MeV. At finite densities, the contribution to the energies of the  $D^+$  and  $D^0$  mesons arising from the Weinberg-Tomozawa term as a function of temperature, is constant, since this term depends on the densities of proton and neutron only, which for a given baryon density and a given value of the isospin asymmetry parameter, are constant. At  $\rho_B = \rho_0$  and  $T=0$ , the Weinberg-Tomozawa term gives a drop of about 23 MeV in the masses of  $D^+$  and  $D^0$  mesons from their vacuum values in isospin symmetric matter and about 31 MeV and 16 MeV for  $D^+$  and  $D^0$  mesons in the isospin asymmetric matter with  $\eta=0.5$ . The mass drop of  $D^+$  meson of 23 MeV in symmetric nuclear matter at zero temperature may be compared

with the value of about 43 MeV, which is the drop in  $D^+$  mass in the Born approximation of the calculation of the coupled channel approach of Ref. [54], when only the Weinberg Tomozawa interaction is included. In the presence of the scalar exchange term as well, the drop in the  $D^+$  mass in the present investigation is seen to be about 144 MeV, which may be compared to the drop in the mass of the  $D^+$  meson with Weinberg-Tomozawa as well as scalar interaction of about 60 MeV in Ref. [54]. In the present investigation, we observe the scalar interaction to be much more dominant than in Ref. [54]. The contributions to the energy of  $D^+$  and  $D^0$  mesons due to the explicit symmetry breaking term (scalar meson exchange term) are observed to increase with temperature upto a particular value of temperature, above which it is seen to decrease with temperature. This is a reflection of the behaviour of the scalar fields  $\sigma$  and  $\zeta$  shown in figures 1 and 2, whose magnitudes are seen to increase with temperature upto a particular value, above which there is seen to be a decrease in their magnitudes. The range term is seen to give an increase in the  $D^+$  and  $D^0$  masses and at  $\rho_B = \rho_0$ , there is seen to be a rise of about 71 MeV at zero temperature for  $\eta=0$ . This value is modified to about 67 MeV at  $T=150$  MeV and about 70 MeV at  $T=175$  MeV. For a density of  $\rho_B = 4\rho_0$  and at zero temperature, the drop in the D-meson masses in the symmetric nuclear matter is seen to be about 27 MeV due to the range term. For this density, the contribution of the  $d_1$  and  $d_2$  terms dominate over the first range term thus leading to an overall drop of the D-meson masses in the medium of about 347 MeV in the symmetric nuclear matter at zero temperature.

It may be noted that the difference in the in-medium energy of  $D^+$  and  $D^-$  mesons arises from Weinberg-Tomozawa term only (see equations (20) and (22)) whereas the scalar meson exchange term and the range terms have identical contributions to the energy of  $D^+$  and  $D^-$  mesons. The repulsive contribution for the  $D^-$  and  $\bar{D}^0$  mesons for the Weinberg-Tomozawa term as compared to the attractive contribution for the  $D^+$  and  $D^0$  mesons, gives a rise in the masses of the  $\bar{D}$  mesons (plotted in figure 6) as compared to the masses of the  $D$  mesons shown in figure 5. At zero density, the contribution from the Weinberg-Tomozawa term is zero and hence the modifications for the  $D^+$  and  $D^-$  are identical, both having negligible changes in their masses due to the cancelling effects of the attractive scalar term and the repulsive range terms. At nuclear matter saturation density, when one takes into

account only the contribution from the Weinberg-Tomozawa term, there is seen to be an increase in the mass of the  $D^-$  meson of about 24 MeV. However, when we consider the contributions due to all the individual terms, the attractive scalar interaction is seen to dominate over the repulsive Weinberg-Tomozawa and the range terms, thereby giving a drop of the  $D^-$  mass of about 27 MeV in the symmetric nuclear matter for  $\rho_B = \rho_0$ . However, for higher densities, the  $d_1$  and  $d_2$  terms of the range term, which are both attractive, give a further drop of the mass of the  $D^-$  meson and for density,  $\rho_B = 4\rho_0$ , the overall drop is seen to be about 162 MeV for  $\eta=0$ .

We study the density dependence of  $D$  and  $\bar{D}$  masses at finite temperatures at selected values of the isospin asymmetric parameter,  $\eta$  and compare the results with the zero temperature case [48]. The isospin symmetric part of the Weinberg-Tomazawa term gives a drop of the  $D$  mass, as can be seen from the expressions of the self-energies of the  $D$  and  $\bar{D}$  mesons given by equations (20) and (22). However, the isospin asymmetric part of this term is seen to give a mass splitting for the  $D^+$  and  $D^0$ , given by the second term of the Weinberg-Tomazawa term, giving a further drop of the  $D^+$  mass, whereas the asymmetry reduces the drop of the mass of  $D^0$ . For the  $\bar{D}$  mesons, the isospin symmetric part of the Weinberg Tomazawa term gives an increase in the mass and the isospin asymmetric contribution of this term gives a further rise in the mass of the  $D^-$  mass, whereas it reduces the increase of the  $\bar{D}^0$  mass in the asymmetric nuclear medium. For both  $D$  and  $\bar{D}$  mesons in the symmetric nuclear matter, the scalar meson interaction is attractive and identical (except for a small difference due to the difference in the vacuum masses of  $D^\pm$  and  $D^0(\bar{D}^0)$  mesons). One might observe from the expressions of the  $D(\bar{D})$  self energies, given by equations (20) and (22) that the non-zero value of  $\delta$  meson arising due to the isospin asymmetry in the medium gives a drop in the masses of  $D^+(D^-)$  in the asymmetric nuclear matter, whereas this interaction is repulsive for  $D^0(\bar{D}^0)$  mesons. The contributions to the  $D$  and  $\bar{D}$  masses due to the range terms are given by the last three terms of the self energies given by equations (20) and (22). The first of these range terms is repulsive, whereas the second and third terms are attractive, when isospin asymmetry is not taken into account. However, due to a nonzero value of the  $\delta$  field arising from isospin asymmetry in the medium, the  $\delta$  term of the first range term leads an increase in the masses of the  $D^+(D^-)$  and a drop in the

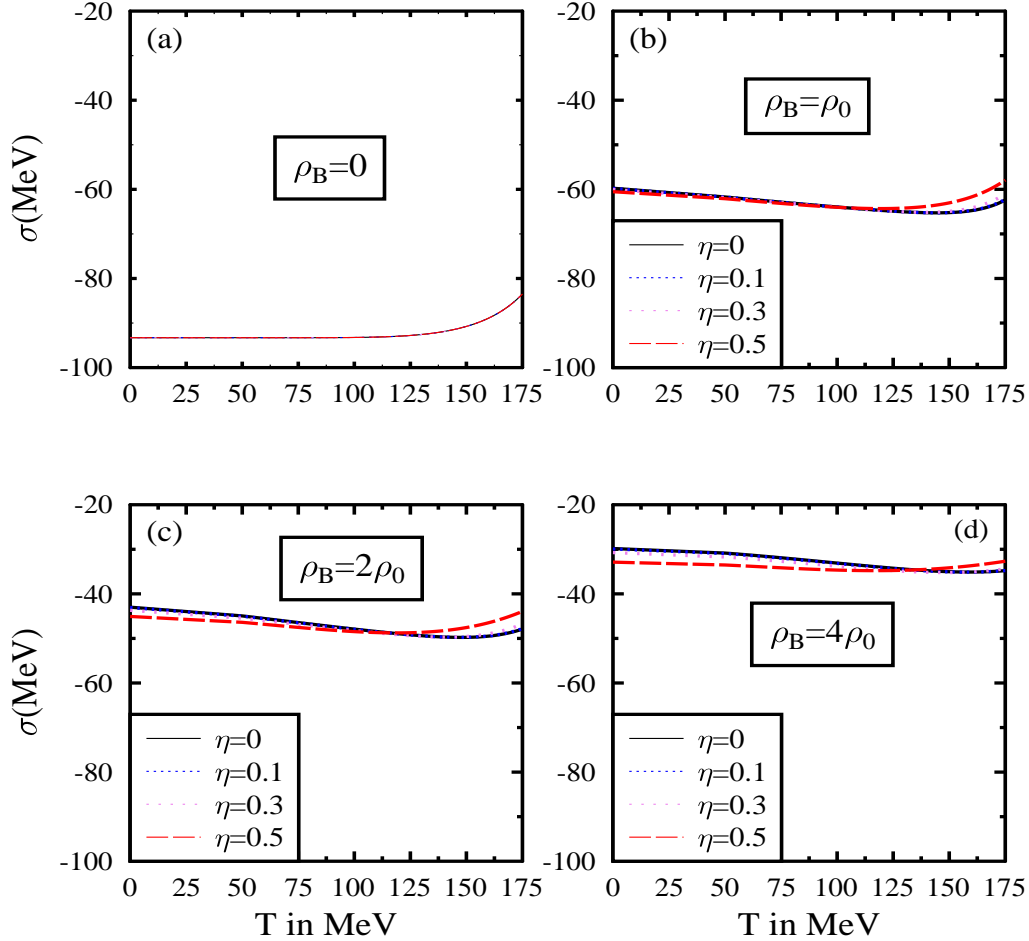


FIG. 1: (Color online) The scalar-isoscalar field  $\sigma$  plotted as a function of temperature at a given baryon density ( $\rho_B = 0, \rho_0, 2\rho_0$  and  $4\rho_0$ ), for different values of the isospin asymmetry parameter,  $\eta$ .

masses of  $D^0(\bar{D}^0)$  mesons. The second of the range terms (the  $d_1$  term) is attractive and gives identical mass drops for  $D^+$  and  $D^0$  in the  $D$  doublet as well as for  $D^-$  and  $\bar{D}^0$  in the  $\bar{D}$  doublet. This term is proportional to  $(\rho_s^p + \rho_s^n)$ , which turns out to be different for the isospin asymmetric case as compared to the isospin symmetric nuclear matter, due to the presence

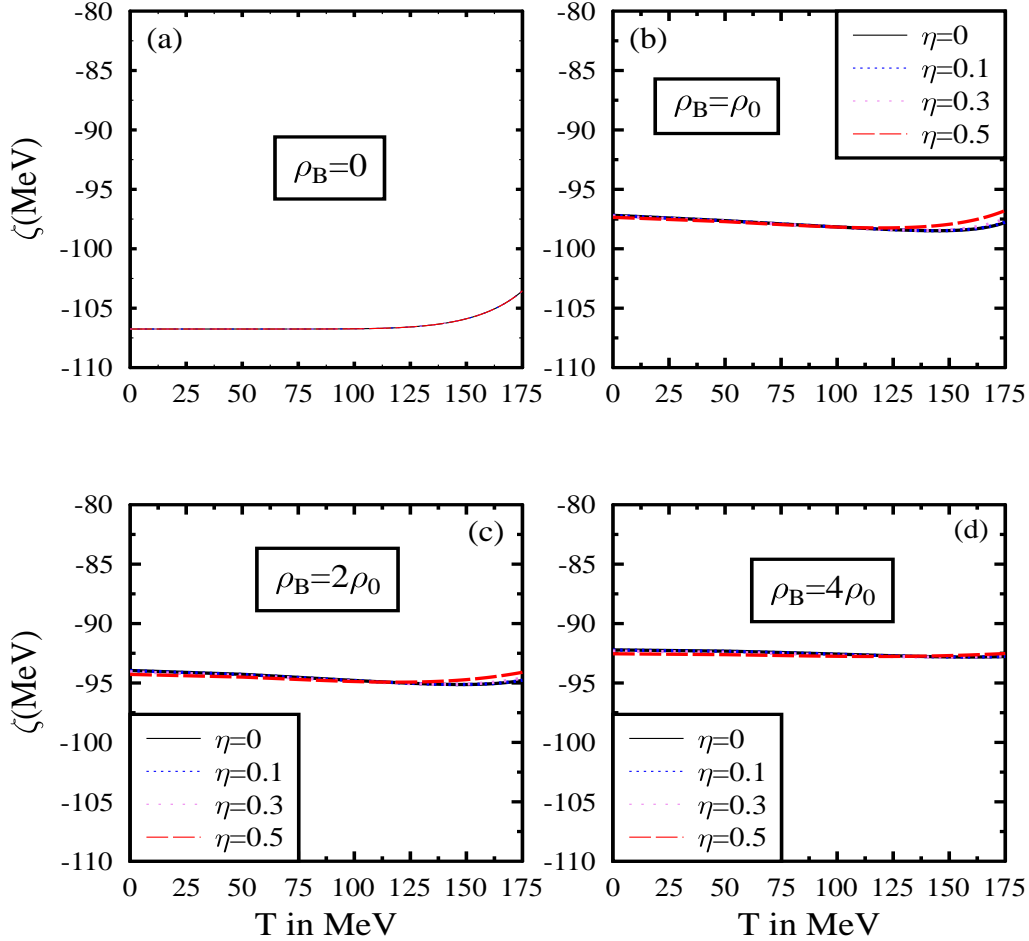


FIG. 2: (Color online) The scalar-isoscalar field  $\zeta$  plotted as a function of temperature at a given baryon density ( $\rho_B = 0, \rho_0, 2\rho_0$  and  $4\rho_0$ ), for different values of the isospin asymmetry parameter,  $\eta$ .

of the  $\delta$  meson. This is because the equations of motion for the scalar fields for the two situations (with/without  $\delta$  mesons) give different values for the mean field,  $\sigma$  ( $\sim (\rho_s^p + \rho_s^n)$ ). The last term of the range term (the  $d_2$  term) has a negative contribution for the energies of  $D^+$  and  $D^0$  mesons as well as for  $D^-$  and  $\bar{D}^0$  mesons for the isospin symmetric matter.



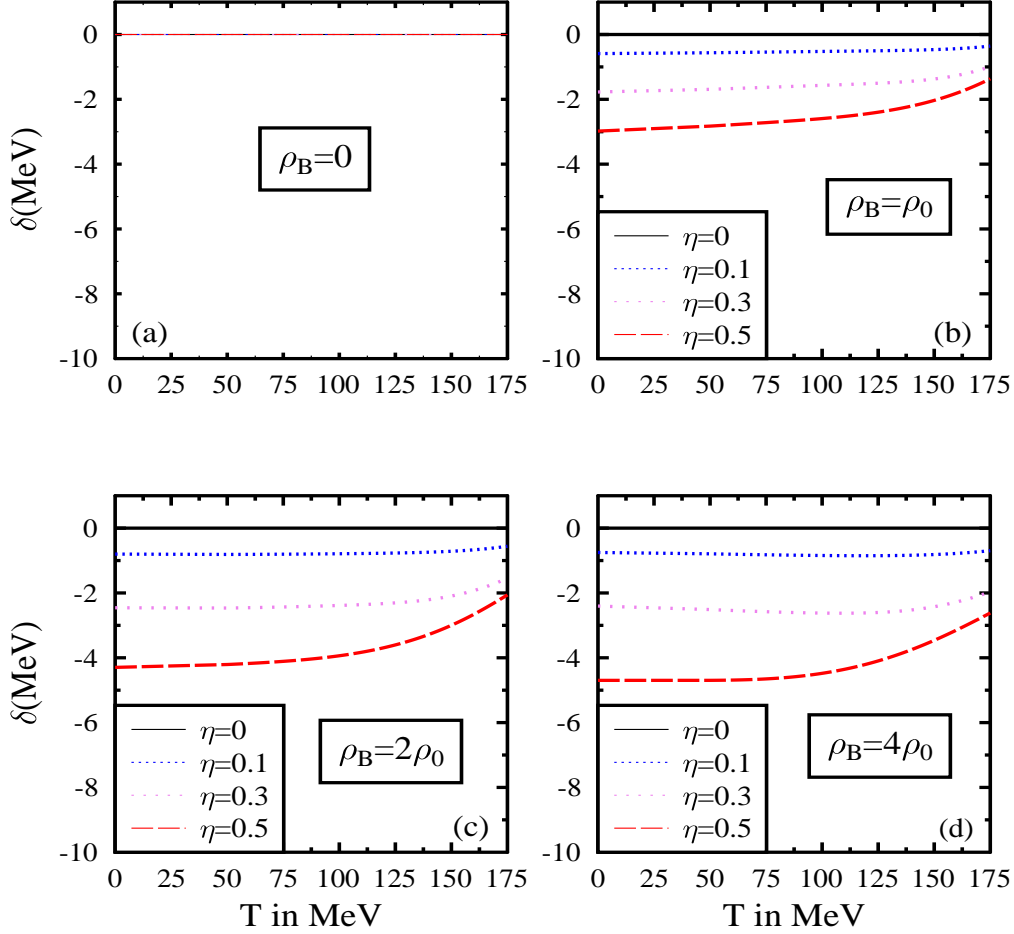


FIG. 3: (Color online) The scalar-isovector field  $\delta$  plotted as a function of temperature at a given baryon density ( $\rho_B = 0, \rho_0, 2\rho_0$  and  $4\rho_0$ ), for different values of the isospin asymmetry parameter,  $\eta$ .

The isospin asymmetric part arising from the  $((\rho_s^n - \rho_s^p))$  term of the  $d_2$  term has a further drop in the masses for  $D^\pm$  mesons, whereas it increases the masses of the  $D^0$  and  $\bar{D}^0$  mesons from their isospin symmetric values. In Fig. 7, we show the variation of the energy of the  $D$  mesons ( $D^+, D^0$ ) at zero momentum with baryon density  $\rho_B$  for different values of isospin

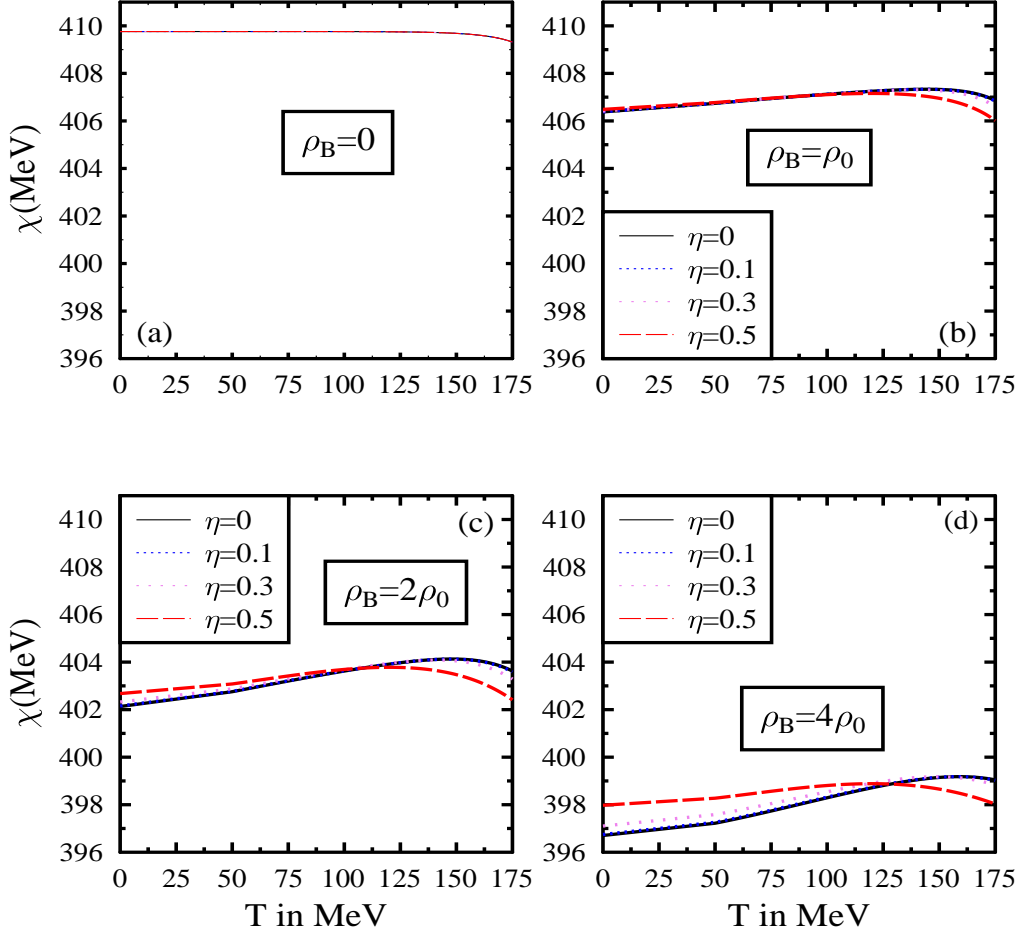


FIG. 4: (Color online) The dilaton field  $\chi$  plotted as a function of temperature at a given baryon density ( $\rho_B = 0, \rho_0, 2\rho_0$  and  $4\rho_0$ ), for different values of the isospin asymmetry parameter,  $\eta$ .

asymmetry parameter  $\eta$  and with the values of temperature as  $T = 0, 100, 150$  MeV. The isospin-asymmetry in the medium is seen to give an increase in the  $D^0$  mass and a drop in the  $D^+$  mass as compared to the isospin symmetric ( $\eta = 0$ ) case. This is observed both for zero temperature [48] and finite temperature cases. At nuclear matter saturation density,  $\rho_B = \rho_0$ , the drop in the mass of  $D^+$  meson from its vacuum value (1869 MeV) is 78 MeV

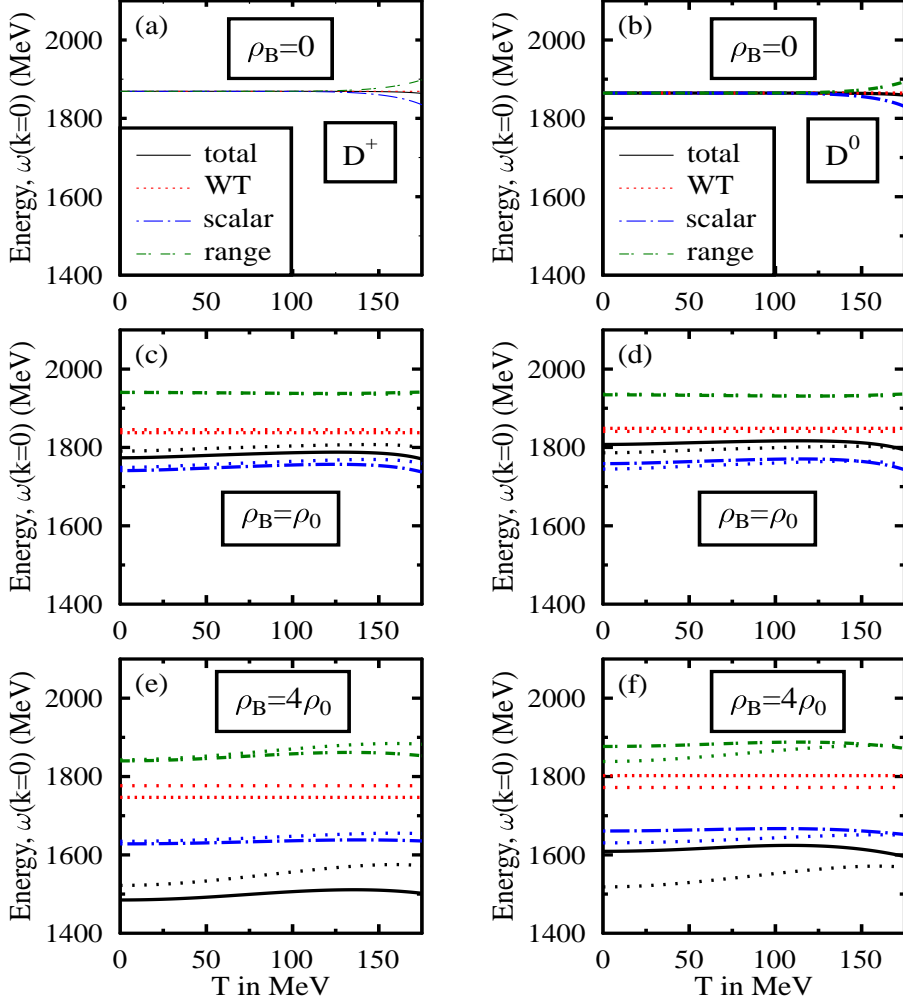


FIG. 5: (Color online) The energies of  $D^+$  meson ((a),(c) and (e)) and of  $D^0$  meson ((b),(d) and (f)), at momentum  $k = 0$ , versus temperature,  $T$ , for different values of the isospin asymmetry parameter ( $\eta = 0$  and  $0.5$ ) and for given values of density ( $\rho_B = 0, \rho_0$  and  $4\rho_0$ ). The values of parameters  $d_1$  and  $d_2$  are calculated from  $KN$  scattering lengths in  $I = 0$  and  $I = 1$  channels.

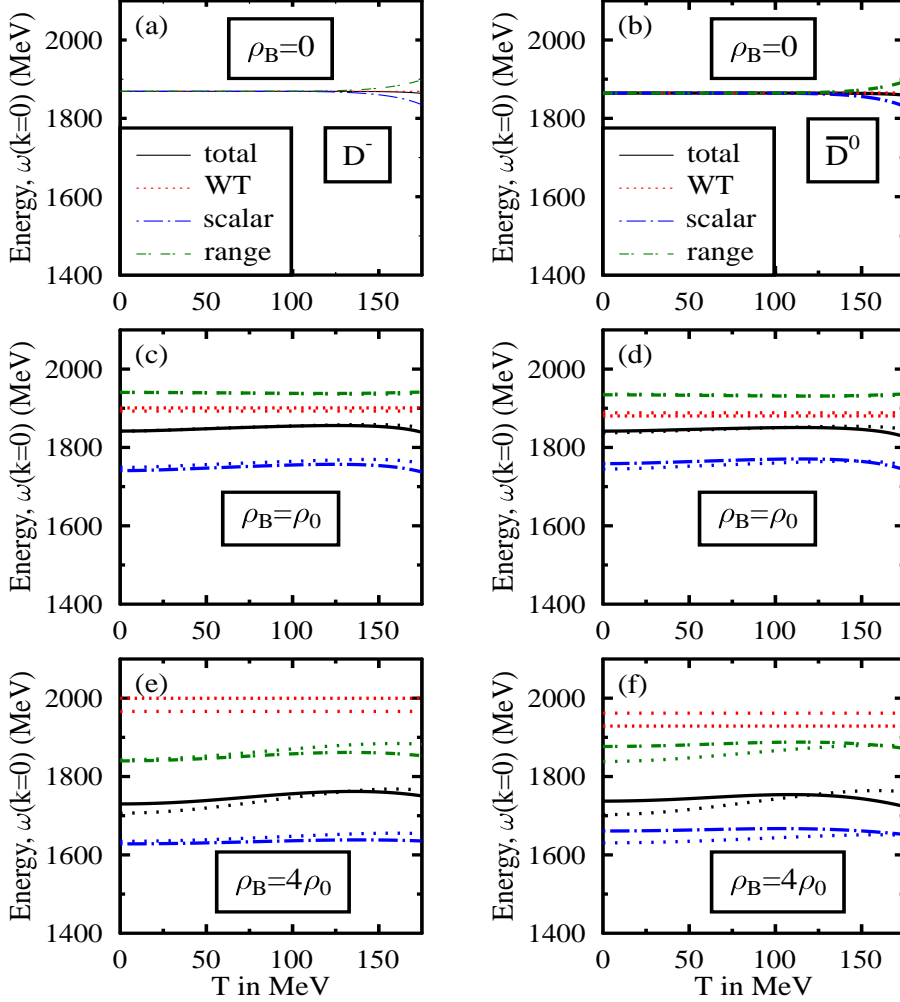


FIG. 6: (Color online) The energies of  $D^-$  meson ((a),(c) and (e)) and of  $\bar{D}^0$  meson ((b),(d) and (f)), at momentum  $k = 0$ , versus temperature,  $T$ , , for different values of the isospin asymmetry parameter ( $\eta = 0$  and  $0.5$ ) and for given values of density ( $\rho_B = 0, \rho_0$  and  $4\rho_0$ ). The values of parameters  $d_1$  and  $d_2$  are calculated from  $KN$  scattering lengths in  $I = 0$  and  $I = 1$  channels.

for zero temperature case in isospin symmetric medium. At a density of  $4\rho_0$ , this drop in the mass of  $D^+$  meson is seen to be about 347 MeV for  $\eta=0$ . At finite temperatures, the drop in the mass of  $D^+$  meson for a given value of isospin asymmetry decreases as compared to the zero temperature case. For example, at nuclear saturation density,  $\rho_0$ , the drop in the mass of  $D^+$  meson turns out to be 72, 65 and 62 MeV at a temperature of  $T = 50, 100$  and  $150$  MeV respectively for the isospin symmetric matter. At a higher density of  $\rho_B = 4\rho_0$ , zero temperature value for the  $D^+$  mass drop of about 347 MeV is modified to 336, 313 and 294 MeV for  $T = 50, 100$  and  $150$  MeV respectively. Thus the masses of  $D$  mesons at finite temperatures and finite densities are observed to be larger than the values at zero temperature case. This is because of the increase in the magnitudes of the scalar fields  $\sigma$  and  $\zeta$  with temperature at finite densities as mentioned earlier. The same behaviour remains for the isospin asymmetric matter. This behaviour of the nucleons and hence of the  $D$ -mesons with temperature was also observed earlier for symmetric nuclear matter at finite temperatures within the chiral effective model [47]. The drop in the mass of  $D^+$  meson is seen to be larger as we increase the value of the isospin asymmetry parameter. We observe that as we change  $\eta$  from 0 to 0.5, then the drop in the mass of  $D^+$  meson is 95 MeV and 384 MeV at densities of  $\rho_0$  and  $4\rho_0$  respectively, for the zero temperature case. At  $T = 50$  MeV these values change to 89 MeV at  $\rho_0$  and 377 MeV at a density of  $4\rho_0$ . For a baryon density,  $\rho_B = \rho_0(4\rho_0)$ , the drop in the  $D^+$  mass is 83 (363) MeV at  $T = 100$  MeV, and 84 (359) MeV at  $T=150$  MeV. We thus observe that for a baryon density of  $\rho_0$ , for  $\eta=0.5$ , the drop in the  $D^+$  mass from its value for  $\eta=0$  is 17, 18 and 22 MeV for  $T=0, 100$  and  $150$  MeV respectively, and for  $\rho_B = 4\rho_0$ , these values are modified to 37, 50 and 65 MeV respectively. Thus we observe that for a given value of density, as we move from  $\eta = 0$  to  $\eta = 0.5$  the drop in mass of  $D^+$  mesons is larger at higher temperatures.

The mass of the  $D^0$  meson drops with density as can be seen from figure 7. The drop in the mass of  $D^0$  meson at density  $\rho_0$  from its vacuum value (1864.5 MeV) is 78, 65, 62 MeV for temperature  $T = 0, 100, 150$  MeV respectively at  $\eta = 0$ . At density  $4\rho_0$  and isospin-asymmetry parameter  $\eta = 0$ , these values become 347, 313 and 294 MeV for temperatures  $T=0, 100$  and  $150$  MeV respectively. For the  $D^0$  meson, there is seen to be an increase in the mass as we move from isospin symmetric to isospin asymmetric medium. For example, at

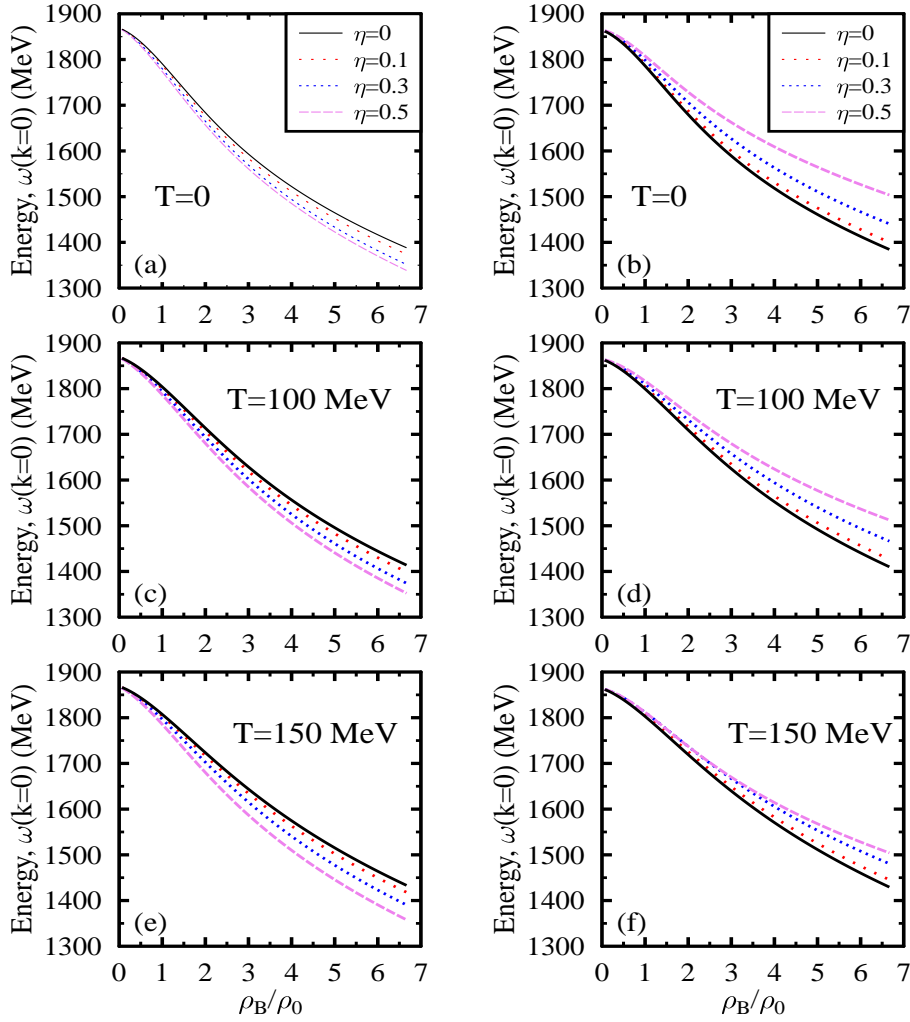


FIG. 7: (Color online) The energies of  $D^+$  meson ((a),(c) and (e)) and of  $D^0$  meson ((b),(d) and (f)), at momentum  $k = 0$ , versus the baryon density (in units of nuclear saturation density),  $\rho_B/\rho_0$ , for different values of the isospin asymmetry parameter ( $\eta = 0, 0.1, 0.3, 0.5$ ) and for given values of temperature ( $T = 0, 100$  MeV and 150 MeV). The values of parameters  $d_1$  and  $d_2$  are calculated from  $KN$  scattering lengths in  $I = 0$  and  $I = 1$  channels.

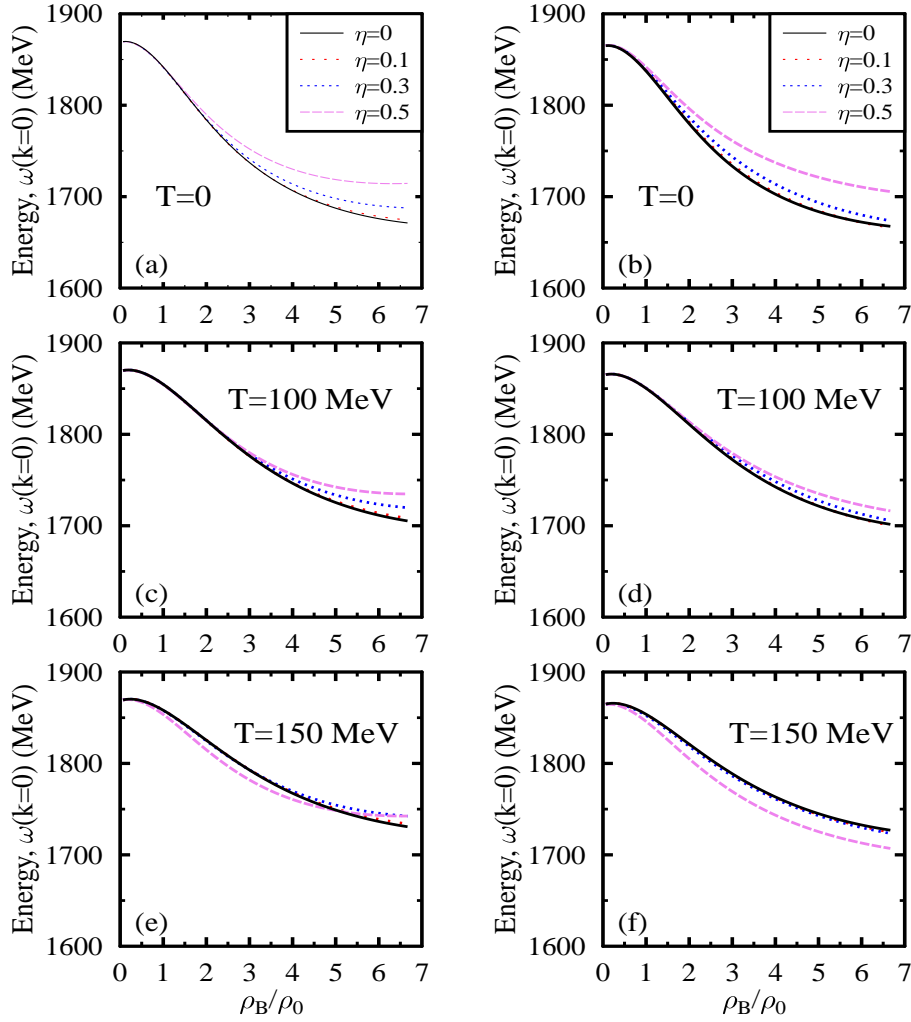


FIG. 8: (Color online) The energies of  $D^-$  meson ((a),(c) and (e)) and of  $\bar{D}^0$  meson ((b),(d) and (f)), at momentum  $k = 0$ , versus the baryon density, expressed in units of nuclear matter saturation density,  $\rho_B/\rho_0$ , for different values of the isospin asymmetry parameter ( $\eta = 0, 0.1, 0.3, 0.5$ ) and for a given temperature ( $T = 0, 100$  MeV and 150 MeV). The parameters  $d_1$  and  $d_2$  are determined from  $KN$  scattering lengths in  $I = 0$  and  $I = 1$  channels.

zero temperature and baryon density equal to  $\rho_0$  and  $4\rho_0$ , the rise in the masses of the  $D^0$  meson are 21 and 91 MeV respectively as we move from isospin-symmetric medium ( $\eta = 0$ ) to isospin-asymmetric medium ( $\eta = 0.5$ ). At a temperature of 100 MeV, these values become 17 MeV at  $\rho_0$  and 72 MeV at  $4\rho_0$ . For  $T=150$  MeV, these values become 9 MeV at  $\rho_0$  and 44 MeV at  $4\rho_0$ . Thus for  $D^0$  mesons, the rise in the mass, is seen to be lowered at higher temperatures as we move from  $\eta = 0$  to  $\eta = 0.5$ . The strong isospin dependence of the  $D^+$  and  $D^0$  meson masses should show up in observables such as their production as well as flow in asymmetric heavy-ion collisions planned at the future facility at FAIR, GSI.

Fig.8 shows the results for the density dependence of the energies of the  $\bar{D}$  mesons at zero momentum at values of the temperature,  $T = 0, 100, 150$  MeV. There is seen to be a drop of the masses of both the  $D^-$  and  $\bar{D}^0$  with density. This is due to the dominance of the attractive scalar exchange contribution as well as the range terms (which becomes attractive above a density of about 2–2.5 times the nuclear matter saturation density) over the repulsive Weinberg-Tomozawa interaction [48]. It is observed that the drop in the mass of  $D^-$  and  $\bar{D}^0$  in isospin symmetric nuclear matter is 27.2 MeV at  $\rho_0$  and 162 MeV at  $4\rho_0$ , at zero temperature, from their vacuum values. As we go to higher temperatures, the drop in the masses of  $D^-$  and  $\bar{D}^0$  mesons decreases. For example, at  $\eta = 0$  and  $\rho_B = \rho_0$  the drop in the mass of  $D^-$  meson is 20.9, 14.3 and 10.8 MeV at a temperature of 50, 100 and 150 MeV respectively. For temperature,  $T=0, 50, 100$  and 150 MeV, at  $\rho_B = \rho_0$ , the values of the drop in the  $D^-$  mass are seen to be 27.2, 21.4, 14.6 and 15.5 MeV respectively, and the drop in the  $\bar{D}^0$  mass are modified from about 27, 21, 14.2 and 11 MeV to 23, 19, 14 and 19 MeV, when  $\eta$  is changed from 0 to 0.5. The masses of  $D^-$  and  $\bar{D}^0$  mesons are observed to have negligible dependence on the isospin asymmetry upto a density of  $\rho_B = \rho_0$ . However, at high densities there is seen to be appreciable dependence of these masses on the parameter,  $\eta$ . As we change  $\eta$  from 0 to 0.5, at  $\rho_B = 4\rho_0$ , the drop in the mass of the  $D^-$  meson is modified from 162 MeV to 139 MeV at zero temperature. At higher temperatures,  $T= 50, 100$  and 150 MeV and at the density of  $4\rho_0$ , when we change  $\eta$  from 0 to 0.5, the values of the  $D^-$  mass drop are modified from 149, 123 and 102 MeV to 128, 111 and 107 MeV respectively. It is seen that, at high densities there is an increase in the masses of both  $D^-$  and  $\bar{D}^0$  mesons in isospin asymmetric medium as compared to those in the isospin



symmetric nuclear matter for temperatures  $T=0, 50$  and  $100$  MeV. However, at  $T = 150$  MeV, it is observed that for densities upto about  $4.5\rho_0$ , the mass of  $D^-$  meson is higher in the isospin symmetric matter as compared to in the isospin asymmetric matter with  $\eta = 0.5$ . It is also seen that the modifications in the masses of  $D^-$  mesons is negligible as we change  $\eta$  from 0 to 0.3 upto a density of about  $4\rho_0$ . For the  $\bar{D}^0$  meson, one sees that the isospin dependence is negligible upto  $\eta=0.3$ . This is because the drop in the mass of  $\bar{D}^0$  mesons due to isospin asymmetry given by Weinberg-Tomozawa term almost cancels with the increase due to the scalar and range terms as we go from  $\eta = 0$  to  $\eta = 0.3$ . At zero temperature [48] as well as for temperatures  $T=50$  and  $100$  MeV, there is seen to be an increase in the mass of the  $\bar{D}$  mesons ( $D^-, \bar{D}^0$ ) as we go from isospin symmetric medium to the isospin asymmetric medium. This is because for  $T=0, 50$  and  $100$  MeV, the increase in mass of  $\bar{D}$  given by the scalar exchange and the range terms dominate over the drop given by the Weinberg Tomozawa term as we go from symmetric nuclear medium ( $\eta = 0$ ) to isospin asymmetric nuclear medium ( $\eta=0.1, 0.3, 0.5$ ). However, at  $T = 150$  MeV, for  $\eta=0.5$ , the drop given by Weinberg term dominates over the rise given by scalar and range terms for  $\bar{D}^0$  and upto a density of about  $4.5\rho_0$  for  $D^-$ , and therefore mass of  $\bar{D}$  meson decreases as we go from symmetric nuclear medium to isospin asymmetric nuclear medium in these density regimes.

The medium modifications of the masses of  $D$  and  $\bar{D}$  mesons in the present investigation are due to the interactions with the nucleons and scalar mesons  $\sigma$ ,  $\zeta$  and  $\delta$  in the hot nuclear medium. The values of the scalar fields in the medium are obtained by solving the equations of motion for the scalar fields and the dilaton field  $\chi$ , given by the coupled equations (9) to (12). The temperature and density dependence of the dilaton field  $\chi$  is seen to be negligible and thus the changes of the values of the scalar fields are observed to be marginal for the present investigation when the medium dependence of the  $\chi$  field is taken into account as compared to when its medium dependence is not taken into account (the so-called frozen glueball approximation where the value of  $\chi$  is taken to be its vacuum value). This leads to the modifications of the  $D$  and  $\bar{D}$  meson masses as marginal as compared to the case when the medium dependence of the dilaton field is not taken into account. For example, for temperature  $T = 0$ , the drop in the mass of  $D^+$  mesons in isospin symmetric

nuclear medium is about 81 MeV and 364 MeV at  $\rho_B = \rho_0$  and  $4\rho_0$  respectively in the frozen glueball approximation, which may be compared to the values of mass drop as 78 MeV and 347 MeV in the present investigation, when we take into account the effect of variation of the dilaton field with density. Hence one observes the difference between them to be marginal, of the order of about 5%. Similarly, the mass drop in the  $D^-$  meson for the isospin asymmetric matter at  $T=0$  in the frozen glueball approximation is observed to be about 30 MeV and 184 MeV at  $\rho_B = \rho_0$  and  $4\rho_0$  respectively. These are different from the values 27 MeV and 162 MeV of the present calculations with medium dependent dilaton field, by about 10%.

In the present calculations, we have used the value of decay constant,  $f_D = 135$  MeV. In isospin symmetry nuclear medium, for temperature  $T=0$ , we observe that the values of mass drops for  $D^+$  and  $D^-$  mesons are about 42 MeV and 4 MeV respectively at nuclear saturation density when we set  $f_D=157$  MeV [78]. These values may be compared to the values of 78 MeV and 27 MeV when the  $f_D$  is taken to be 135 MeV. Hence by modifying the value of  $f_D$  by about 15% leads to modifications of the mass shifts of  $D^+$  and  $D^-$  by about 46% and 85% at density,  $\rho_0$  for symmetric nuclear matter at zero temperature. The mass drops for the value of the density as  $4\rho_0$  for symmetric nuclear matter at  $T=0$ , are modified from 347 MeV and 162 MeV to the values 238 MeV and 93 MeV for the  $D^+$  and  $D^-$  respectively, when we change the value of  $f_D$  from 135 MeV to 157 MeV. Hence, for  $\rho_B = 4\rho_0$ , the modifications for the  $D^+(D^-)$  masses are about 30% and 40% when we change the value of the D-meson decay constant. Hence, there seems to be appreciable dependence of the mass shifts of the  $D(\bar{D})$  mesons with the D-meson decay constant.

We next examine how the masses of the  $D$  and  $\bar{D}$  mesons change if we determine the parameters  $d_1$  and  $d_2$  from DN scattering lengths calculated to be  $-0.43$  fm and  $-0.41$  fm in the  $I = 0$  and  $I = 1$  channels respectively, in a coupled channel approach [56]. In figures 9 and 10, we show the variation of the energies of  $D$  and  $\bar{D}$  mesons respectively, at zero momentum, with baryon density  $\rho_B$  for different values of the isospin asymmetry parameter  $\eta$  and for the values of the temperature as  $T = 0, 100, 150$  MeV. The values of  $d_1$  and  $d_2$  parameters determined from these values of the DN scattering lengths turn out to be  $8.95/m_D$  and  $0.52/m_D$  respectively, which can be expressed in terms of the mass of the kaon

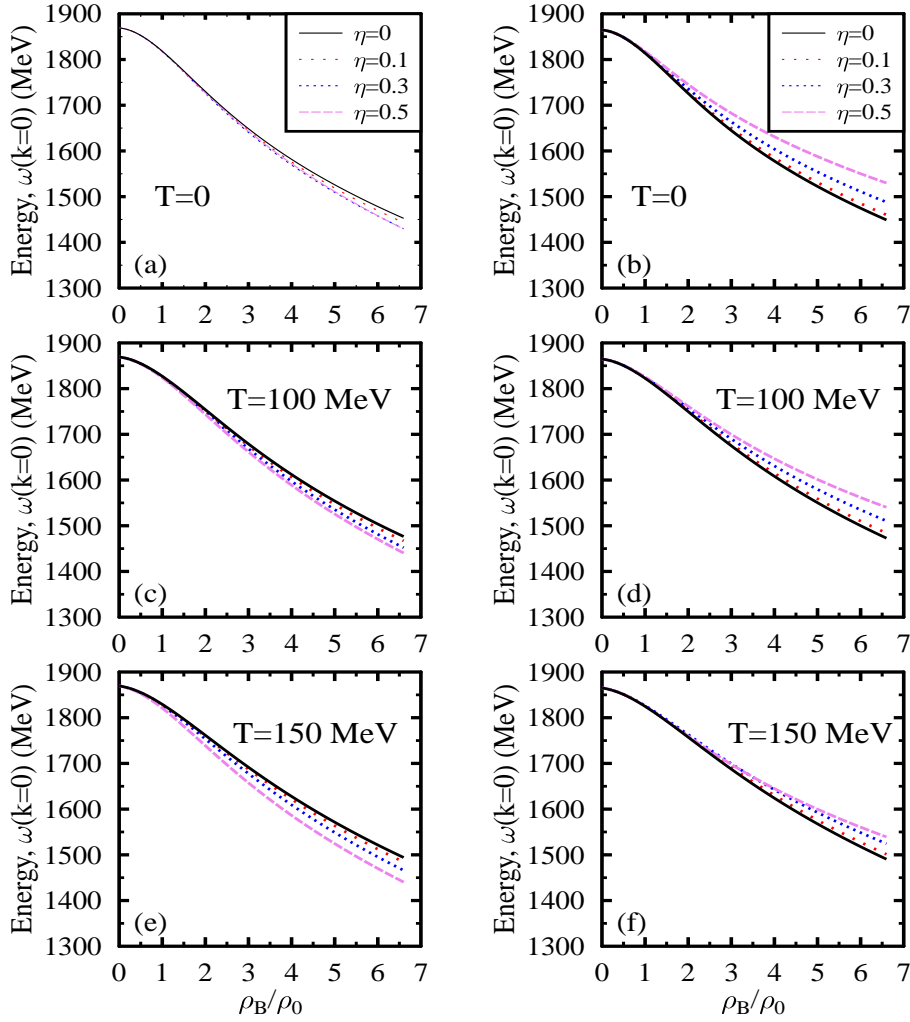


FIG. 9: (Color online) The energies of  $D^+$  meson ((a),(c) and (e)) and of  $D^0$  meson ((b),(d) and (f)), at momentum  $k = 0$ , versus the baryon density (in units of nuclear saturation density),  $\rho_B/\rho_0$ , for different values of the isospin asymmetry parameter ( $\eta = 0, 0.1, 0.3, 0.5$ ) and for given values of temperature ( $T = 0, 100$  MeV and 150 MeV). The values of parameters  $d_1$  and  $d_2$  are calculated from  $DN$  scattering lengths in  $I = 0$  and  $I = 1$  channels.

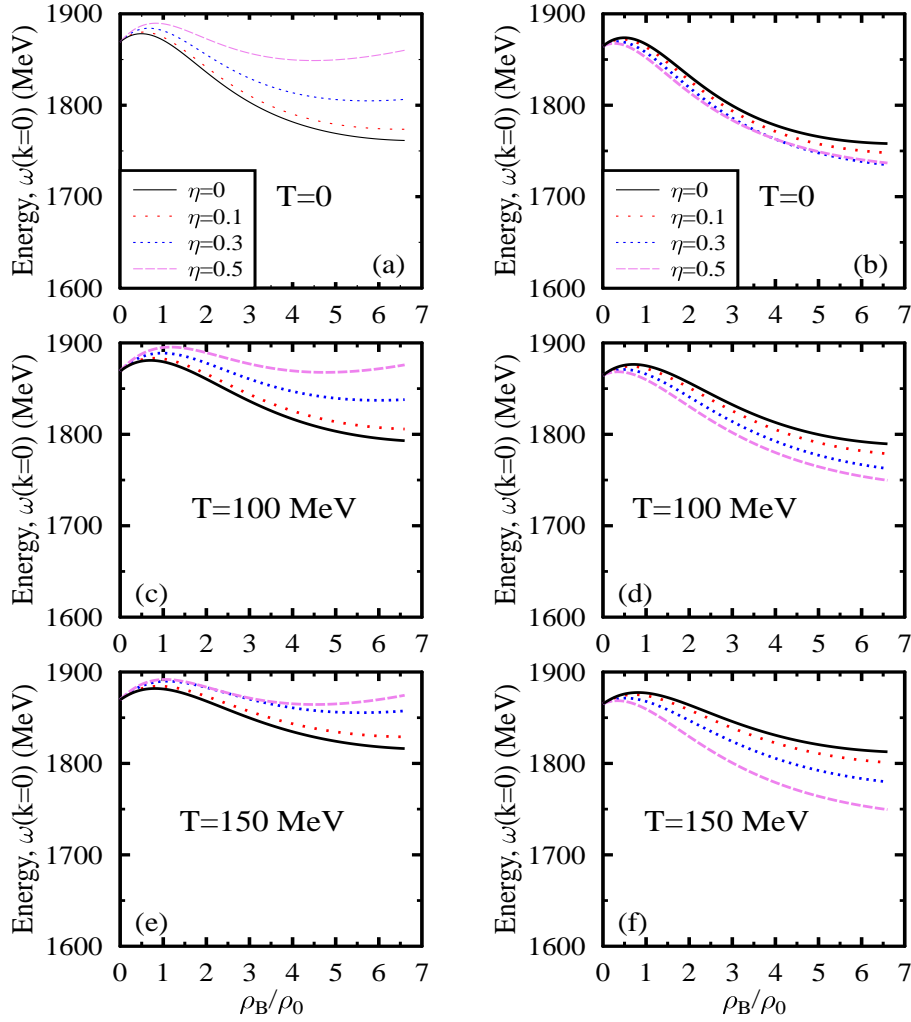


FIG. 10: (Color online) The energies of  $D^-$  meson ((a),(c) and (e)) and of  $\bar{D}^0$  meson ((b),(d) and (f)), at momentum  $k = 0$ , versus the baryon density, expressed in units of nuclear saturation density,  $\rho_B/\rho_0$ , for different values of the isospin asymmetry parameter ( $\eta = 0, 0.1, 0.3, 0.5$ ) and for given values of temperature ( $T = 0, 100$  MeV and  $150$  MeV). The values of parameters  $d_1$  and  $d_2$  are calculated from  $DN$  scattering lengths in  $I = 0$  and  $I = 1$  channels.

as  $2.385/m_K$  and  $0.14/m_K$ . These values of parameters are smaller than the values of  $d_1$  and  $d_2$  as  $2.56/m_K$  and  $0.73/m_K$  respectively, when determined from the KN scattering lengths in  $I = 0$  and  $I = 1$  channels. Since both of these terms are attractive, the masses of the  $D$  ( $\bar{D}$ ) mesons turn out to have a smaller drop when these parameters are fitted from DN scattering lengths as compared to when these are fitted from the KN scattering lengths.

With the set of values of  $d_1$  and  $d_2$  parameters as fitted from the DN scattering lengths, the drop in the masses of  $D^+$  meson in the isospin symmetric nuclear medium at nuclear matter saturation density  $\rho_0$  turns out to be 51, 42 and 40 MeV, at the values of temperature as,  $T = 0, 100$  and  $150$  MeV respectively. These may be compared to the values of the mass drop of  $D^+$  meson of 78, 65 and 62 MeV for  $T=0, 100$  and  $150$  MeV, respectively, when the parameters are fitted from the KN scattering lengths. For  $D^0$  mesons the values of mass drop are 50, 42 and 40 MeV at  $T=0, 100$  and  $150$  MeV respectively, when  $d_1$  and  $d_2$  are fitted from the DN scattering lengths. These may be compared to the values of mass drop of  $D^0$  meson of about 78, 65 and 62 MeV, when these are fitted to the KN scattering lengths. At higher densities, the  $d_1$  and  $d_2$  terms become more dominant and overcome the repulsive interaction of the first range term, leading to a drop of the D-meson masses due to the range term as well. At a density of  $\rho_B = 4\rho_0$ , the drop in the  $D^+$  mass is observed to be about 286 MeV, 256 MeV, 240 MeV for  $T=0, 100$  MeV and  $150$  MeV, which may be compared to the values of 347 MeV, 313 MeV and 294 MeV for  $T=0, 100, 150$  MeV, when the parameters  $d_1$  and  $d_2$  are fitted from the KN scattering lengths.

We plot the masses of the  $\bar{D}$  mesons in figure 10, with the values of  $d_1$  and  $d_2$  are fitted from DN scattering lengths [56]. At the nuclear saturation density  $\rho_0$ , the value of mass of  $D^-(\bar{D}^0)$  meson in isospin symmetric nuclear medium, is observed to increase by 2, 10 (10.5) and 12 (12.5) MeV at  $T = 0, 100$  and  $150$  MeV, respectively. However, at higher densities, the  $d_1$  and  $d_2$  terms become more dominant thus leading to a drop of the  $D^-(\bar{D}^0)$  masses in the nuclear matter. For  $\rho_B = 4\rho_0$ , the mass of  $D^-(\bar{D}^0)$  meson is seen to decrease by 87 (86) MeV, 52 (51.5) MeV and 34 (33.6) MeV respectively. These may be compared to the results of the in-medium masses of  $D^-(\bar{D}^0)$  meson, when  $d_1$  and  $d_2$  are fitted from the KN scattering lengths. In the latter case, as already mentioned for  $T=0, 100$  MeV and  $150$  MeV and for  $\rho_B = 4\rho_0$ , there is seen to be a drop of  $D^-(\bar{D}^0)$  mass of 27.2 (27.2) 14.3 (14.2) and

10.8 (10.8) MeV at  $\rho_B = \rho_0$  and 162 (162) 122.6 (122) and 101.7 (101.2) MeV at  $\rho_B = 4\rho_0$ .

As mentioned earlier, the in-medium mass of  $D^+$  meson decreases with increase in the isospin asymmetry of the nuclear medium. However, with values of  $d_1$  and  $d_2$  parameters fitted from DN scattering lengths, the decrease in the mass with isospin asymmetry of the nuclear medium is observed to be smaller than that when these parameters are fitted from the KN scattering lengths. For example, at  $\rho_B = 4\rho_0$  and  $T = 0$ , as we move from  $\eta = 0$  to  $\eta = 0.3$ , the in-medium masses of  $D^+$  mesons decrease by 27 MeV with the values from the KN scattering lengths and by 13 MeV with the set of values of  $d_1$  and  $d_2$  parameters fitted from the DN scattering lengths. For the  $D^+$  meson, the  $d_1$  term gives a rise in the mass whereas the  $d_2$  term gives drop in the mass when one has a nonzero value of the asymmetry parameter as compared to the symmetric nuclear matter. Also, the first range term gives an increase due to isospin asymmetry, whereas both the Weinberg Tomozawa term and scalar meson exchange term give drop in the mass of the  $D^+$  meson. Due to the drop arising from the  $d_2$  term, Weinberg Tomozawa term and the scalar term dominating over the increase due to the  $d_1$  term, the mass of  $D^+$  meson decreases in the isospin asymmetric nuclear medium as compared to the mass in symmetric matter. When we use the values of the parameters  $d_1$  and  $d_2$  as calculated from the DN scattering lengths, then there is a smaller contribution to the drop of the mass due to  $d_2$  term, as compared to the drop due to isospin asymmetry arising to this term when the parameters  $d_1$  and  $d_2$  are calculated from the KN scattering lengths. This is due to the smaller value for the  $d_2$  in the former case. Therefore, with values of  $d_1$  and  $d_2$  parameters calculated from the DN scattering lengths, the drop in the mass of  $D^+$  meson with isospin asymmetry of the medium is seen to be small.

With the set of values of the parameters  $d_1$  and  $d_2$  as fitted from the KN scattering lengths, the mass of the  $D^-$  meson is observed to increase with the isospin asymmetry of the medium. With the values of  $d_1$  and  $d_2$  parameters as fitted from the DN scattering lengths, there is seen to be larger increase in the mass of the  $D^-$  meson due to the isospin asymmetry of the nuclear medium. For example, at  $\rho_B = 4\rho_0$  and  $T = 0$ , as we move from  $\eta = 0$  to  $\eta = 0.3$ , the mass of the  $D^-$  meson is seen to increase by about 7 MeV when we use the values of  $d_1$  and  $d_2$  as fitted from the KN scattering lengths and by 32 MeV with

the values of  $d_1$  and  $d_2$  fitted from the DN scattering lengths. For the  $D^-$  meson, there is an increase in the mass due to isospin asymmetry due to the Weinberg Tomozawa term, the first range term (term with coefficient  $(-\frac{1}{f_D})$ ) and the  $d_1$  term, whereas the scalar term and  $d_2$  term give drop in the mass of  $D^-$  meson as compared to the symmetric nuclear matter. The net effect is that the mass of the  $D^-$  meson increases with the isospin asymmetry of the nuclear medium. With the values of  $d_1$  and  $d_2$  fitted from DN scattering lengths, because of the smaller value of  $d_2$ , the drop arising from the  $d_2$  term due to isospin asymmetry in the medium is observed to be smaller than the case when the parameters  $d_1$  and  $d_2$  are fitted from the KN scattering lengths (same as for  $D^+$  mesons). There is seen to be larger increase in the in-medium masses of  $D^-$  mesons as a function of the isospin asymmetry of the nuclear medium when the parameters are determined from the DN scattering lengths as compared to the KN scattering lengths.

With the values of  $d_1$  and  $d_2$  calculated from KN scattering lengths, the mass of the  $\bar{D}^0$  meson increases with the isospin asymmetry  $\eta$  of the medium as shown in figure 8. However, if we use the values of  $d_1$  and  $d_2$ , fitted from the DN scattering lengths, then the mass of  $\bar{D}^0$  mesons are seen to decrease with increase in the isospin asymmetry of the medium as shown in the figure 10. For example, at  $\rho_B = 4\rho_0$  and  $T = 0$ , as we move from  $\eta = 0$  to  $\eta = 0.3$ , the  $\bar{D}^0$  mass increases by about 10 MeV when the parameters  $d_1$  and  $d_2$  are fitted from the KN scattering length, whereas the mass of  $\bar{D}^0$  is seen to decrease by about 16 MeV when  $d_1$  and  $d_2$  are calculated from the DN scattering lengths. The reason is, as a function of isospin asymmetry of the medium the  $d_1$  and  $d_2$  terms give rise to the masses of  $\bar{D}^0$  mesons and the first range term gives a drop in the  $\bar{D}^0$  mass. The values of  $d_1$  and  $d_2$  are larger when fitted from the KN scattering lengths as compared to when calculated from the DN scattering lengths. Hence in the former case, the the increase in  $\bar{D}^0$  from isospin asymmetry arising from the  $d_1$  and  $d_2$  terms dominates over the drop given by first range term. Therefore, with isospin asymmetry of the medium the mass of  $\bar{D}^0$  increases in the former situation. However, for the values of  $d_1$  and  $d_2$  parameters fitted from DN scattering lengths, the increase due to  $d_1$  and  $d_2$  terms is dominated by first range term. The Weinberg term gives a drop and the scalar term gives an increase in the mass of  $\bar{D}^0$  meson with the isospin asymmetry of the nuclear medium. The net effect on the mass of the  $\bar{D}^0$  meson is a

drop with isospin asymmetry,  $\eta$ , of the nuclear medium.

The parameters  $d_1$  and  $d_2$  have the same effect on the masses of the  $D^0$  and  $\bar{D}^0$  mesons of giving an increase in their mass in the isospin asymmetric medium as compared to the masses in the symmetric nuclear matter whereas the first range term gives a drop in their masses. The Weinberg Tomozawa term and the scalar meson exchange term lead to an increase in the mass of  $D^0$  meson with isospin asymmetry. As can be seen from figures 7 and 9, the net effect on the  $D^0$  meson mass is an increase with isospin asymmetry, but the rise is less for the case when  $d_1$  and  $d_2$  are calculated from the DN scattering lengths due to the smaller values of  $d_1$  and  $d_2$  as compared to when these parameters are determined from the KN scattering lengths. For example, at  $\rho_B = 4\rho_0$  and  $T = 0$ , as we move from  $\eta = 0$  to  $\eta = 0.3$ , the in-medium mass of  $D^0$  mesons increases by about 45 MeV when the parameters are calculated from KN scattering lengths and by about 26 MeV when fitted from the DN scattering lengths.

The mass modifications of  $D$ -mesons at finite density have been studied in the QCD sum rule and the mass shift at nuclear matter saturation density was found to be about  $-50$  MeV [37]. In the QMC model, the mass shift was around  $-60$  MeV [39]. In the present investigation, at finite densities the magnitude of the scalar fields first increases with increase in the temperature upto a particular value of temperature after which it starts decreasing. This behaviour is then reflected in the variation of the nucleon mass with temperature at finite densities. In QMC model, the behaviour of the scalar field is also seen to be the same as in the present model [79]. However, in the QMC model, the nucleon mass is seen to monotonically rise with temperature and there is no change in this trend observed even upto a temperature of about 250 MeV [79] unlike in the present chiral model or in the Walecka model [77]. This is because  $\sigma$  field in the QMC model is not as strong as in the chiral model or the Walecka model. In QMC model, there are contributions to the masses of nucleons from the thermal excitations of the quarks inside the nucleon bag. This contribution of quarks dominates over the  $\sigma$  field in QMC model [79]. The small attractive mass shift for the  $\bar{D}$  mesons, obtained within our calculations are in favor of charmed mesic nuclei as suggested in the QMC model [39]. This is, however, contrary to a repulsive potential obtained for the  $\bar{D}$  mesons in the coupled channel approach [55]. In our investigation, if we



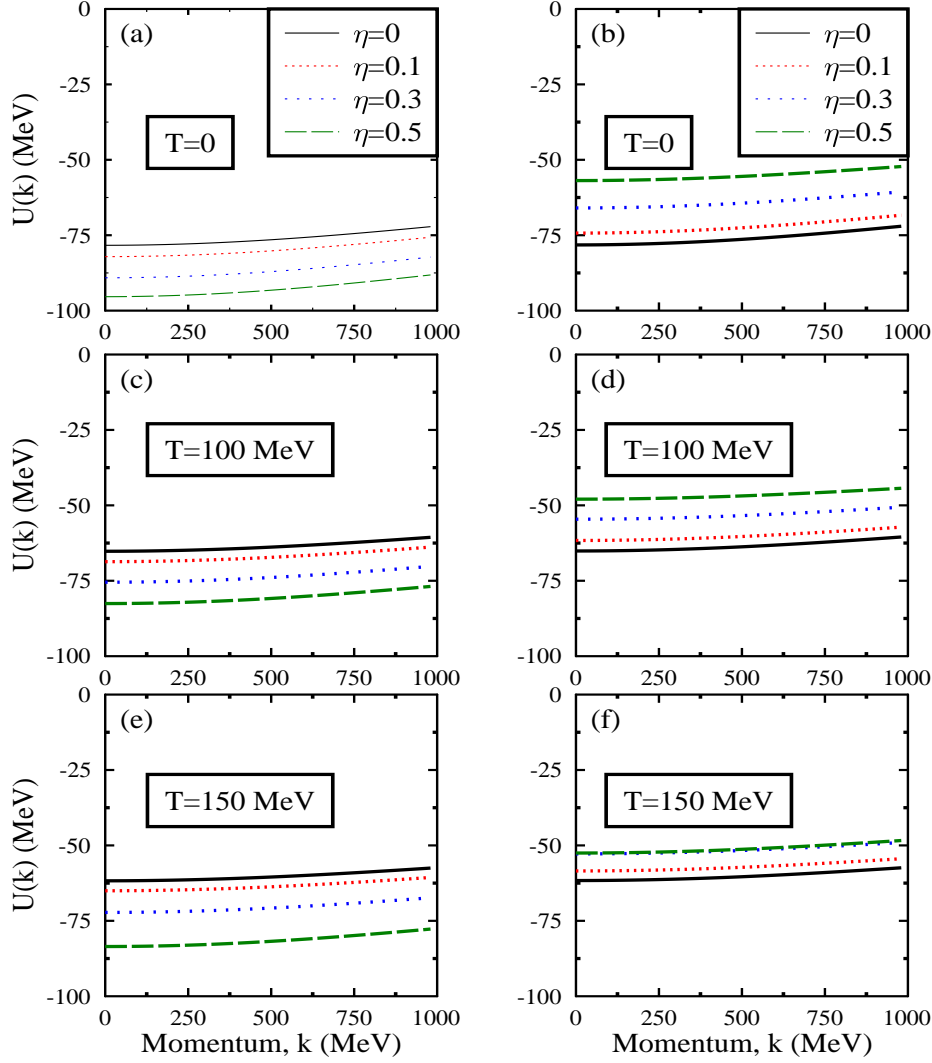


FIG. 11: (Color online) The optical potential of  $D^+$  meson ((a),(c) and (e)) and of  $D^0$  meson ((b),(d) and (f)), are plotted as functions of momentum for  $\rho_B = \rho_0$ , for different values of the isospin asymmetry parameter ( $\eta = 0, 0.1, 0.3, 0.5$ ) and for given values of temperature ( $T = 0, 100$  MeV and 150 MeV). The values of parameters  $d_1$  and  $d_2$  are calculated from  $KN$  scattering lengths in  $I = 0$  and  $I = 1$  channels.

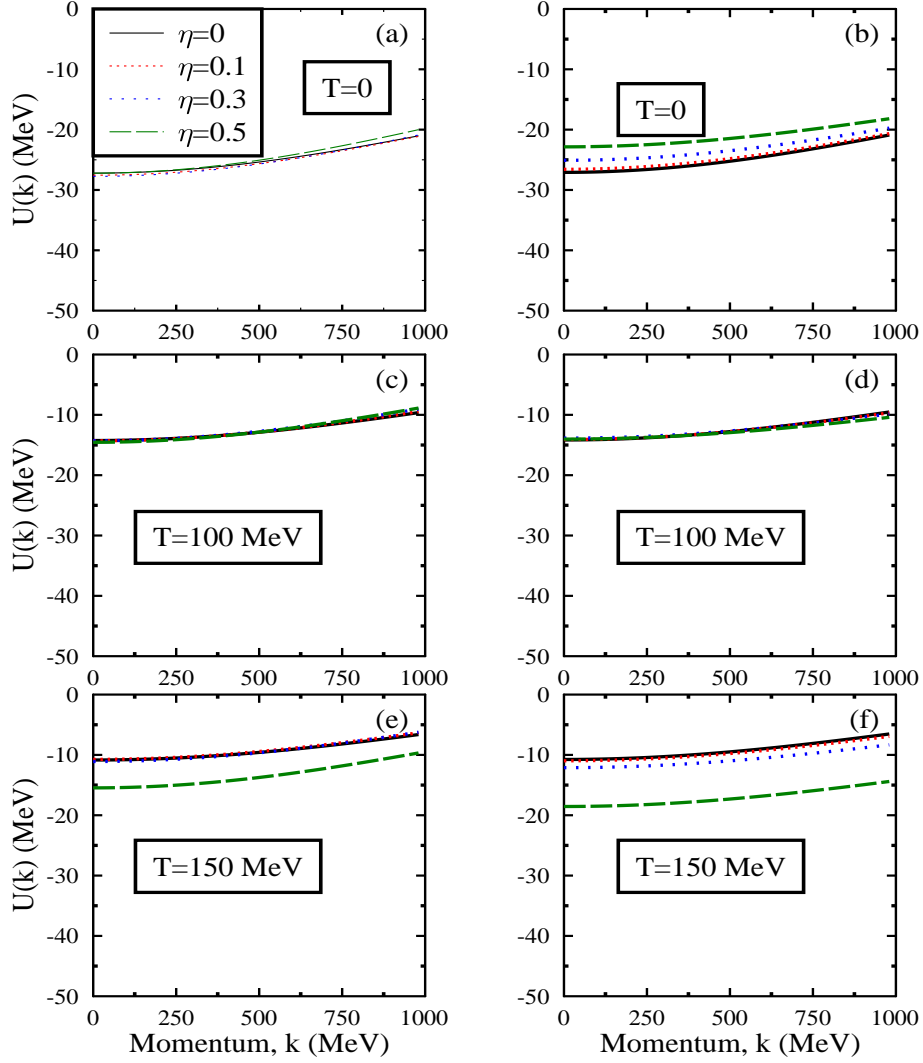


FIG. 12: (Color online) The optical potential of  $D^-$  meson ((a),(c) and (e)) and of  $\bar{D}^0$  meson ((b),(d) and (f)), are plotted as functions of momentum for  $\rho_B = \rho_0$ , for different values of the isospin asymmetry parameter ( $\eta = 0, 0.1, 0.3, 0.5$ ) and for given values of temperature ( $T = 0, 100$  MeV and 150 MeV). The values of parameters  $d_1$  and  $d_2$  are calculated from  $KN$  scattering lengths in  $I = 0$  and  $I = 1$  channels.

do not take into consideration the effect of the range terms on the in-medium properties of  $D$  mesons then at nuclear saturation density  $\rho_0$  and temperature  $T = 0$ , the mass of  $D^+$  and  $D^-$  mesons drop by 144 MeV and 96 MeV respectively in isospin symmetric nuclear medium ( $\eta = 0$ ). The large mass-shift shows the absence of repulsion due to total range term at nuclear saturation density. However, in coupled channel approach of Ref.[55], a repulsive mass-shift of 11 MeV is given for  $\bar{D}$  mesons. Figures 11 and 13 show the isospin dependence of the optical potentials for the  $D$  mesons as functions of the momentum, for densities  $\rho_0$  and  $4\rho_0$  respectively and for values of the temperature as  $T = 0, 100, 150$  MeV. These optical potentials are plotted for the values of  $d_1$  and  $d_2$  calculated from the KN scattering lengths. Figures 12 and 14 illustrate the optical potentials for the  $\bar{D}$  doublet. The isospin dependence of optical potentials is seen to be quite significant for high densities for the D-meson doublet ( $D^+, D^0$ ) as compared to those for the  $\bar{D}$  doublet. This is a reflection of the strong isospin dependence of the masses of the D-mesons as compared to the  $\bar{D}$  as has been already illustrated in figures 7 and 8. For the  $\bar{D}$  mesons, it is seen, from figure 8, that the masses of the the  $D^-$  meson and  $\bar{D}^0$  meson for a fixed value of the isospin asymmetry parameter,  $\eta$  are very similar, an observation which was seen earlier for the zero temperature case [48]. These are reflected in their optical potentials, plotted in figures 12 and 14, where one sees a maximum difference of about 5 MeV or so between  $D^-$  and  $\bar{D}^0$  for  $\rho_B = \rho_0$  and about 10 – 15 MeV for  $\rho_B = 4\rho_0$ . The present investigations of the optical potentials for the  $D$  and  $\bar{D}$  mesons show a much stronger dependence of isospin asymmetry on the  $D$  meson doublet, as compared to that in the  $\bar{D}$  meson doublet, as was already observed for the zero temperature case. However, when the parameters  $d_1$  and  $d_2$  are fitted from the DN scattering lengths, then one sees a greater sensitivity to the isospin asymmetry in the masses of the  $\bar{D}$  mesons as compared to the masses of the  $D$  mesons, as illustrated in figures 9 and 10.

We shall now investigate how the behaviour of the dilaton field  $\chi$  in the hot asymmetric nuclear matter affects the in-medium masses of the charmonium states  $J/\psi, \psi(3686)$  and  $\psi(3770)$ . In figures 15, 16 and 17, we show the shifts of the masses of charmonium states  $J/\psi, \psi(3686)$  and  $\psi(3770)$  from their vacuum values, as functions of the baryon density for given values of temperature  $T$  and for different values of the isospin asymmetry parameter,

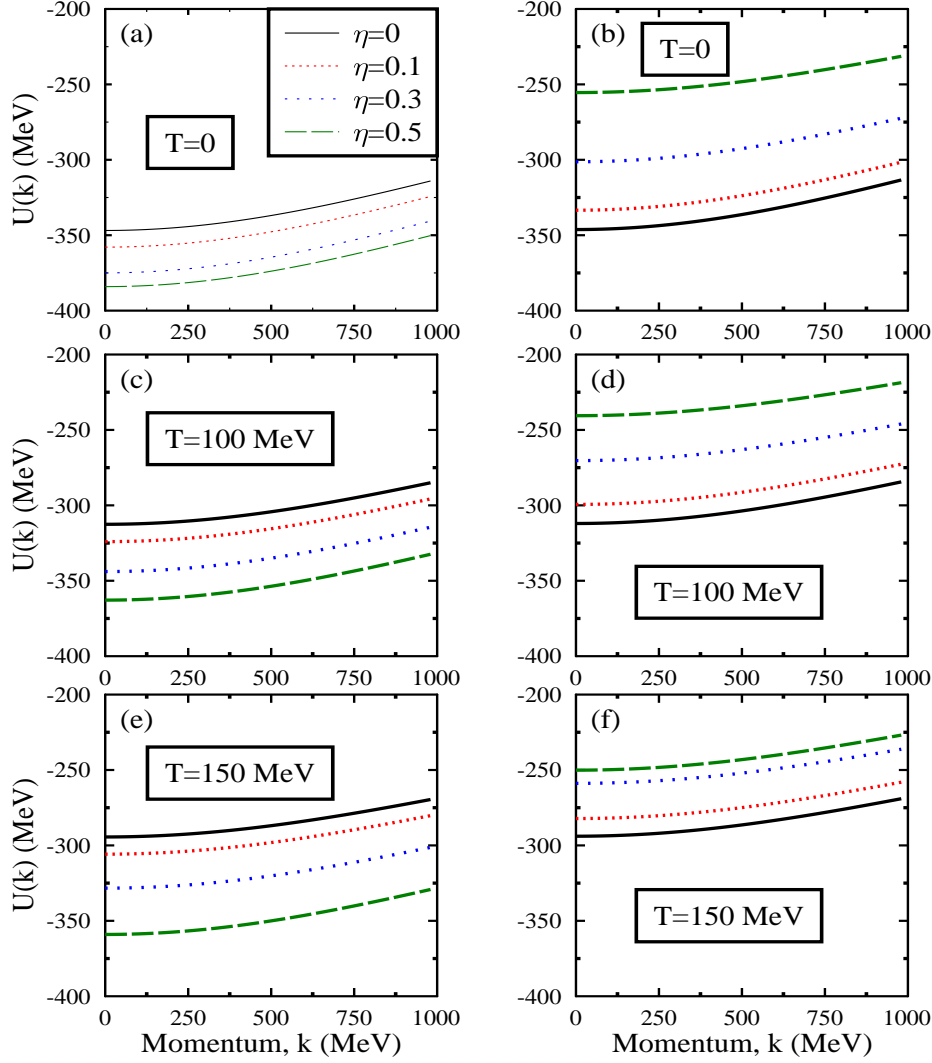


FIG. 13: (Color online) The optical potential of  $D^+$  meson ((a),(c) and (e)) and of  $D^0$  meson ((b),(d) and (f)), are plotted as functions of momentum for  $\rho_B = 4\rho_0$ , for different values of the isospin asymmetry parameter ( $\eta = 0, 0.1, 0.3, 0.5$ ) and for given values of temperature ( $T = 0, 100$  MeV and 150 MeV). The values of parameters  $d_1$  and  $d_2$  are calculated from  $KN$  scattering lengths in  $I = 0$  and  $I = 1$  channels.

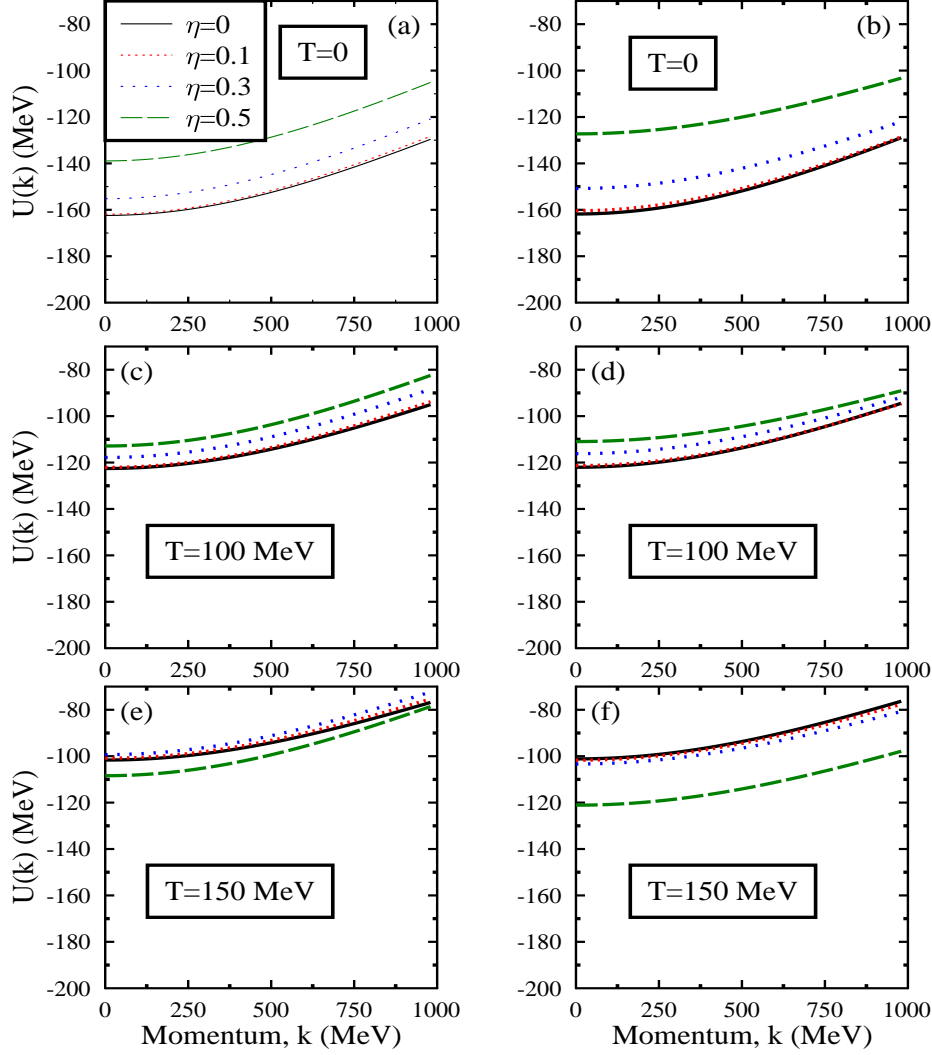


FIG. 14: (Color online) The optical potential of  $D^-$  meson ((a),(c) and (e)) and of  $\bar{D}^0$  meson ((b),(d) and (f)), are plotted as functions of momentum for  $\rho_B = 4\rho_0$ , for different values of the isospin asymmetry parameter ( $\eta = 0, 0.1, 0.3, 0.5$ ) and for given values of temperature ( $T = 0, 100$  MeV and 150 MeV). The values of parameters  $d_1$  and  $d_2$  are calculated from  $KN$  scattering lengths in  $I = 0$  and  $I = 1$  channels.

$\eta$ . We have shown the results for the values of the temperature,  $T = 0, 50, 100$  and  $150$  MeV. At the nuclear matter saturation density,  $\rho_B = \rho_0$  at temperature  $T = 0$ , the mass-shift for  $J/\psi$  meson is  $-8.6$  MeV in the isospin symmetric nuclear medium ( $\eta = 0$ ) and in the asymmetric nuclear medium, with isospin asymmetry parameter  $\eta = 0.5$ , it is seen to be about  $-8.4$  MeV. For  $\rho_B = 4\rho_0$  and at zero temperature, the mass-shift for  $J/\psi$  meson is observed to be about  $-32.2$  MeV in the isospin symmetric nuclear medium ( $\eta = 0$ ) and in isospin asymmetric nuclear medium ( $\eta = 0.5$ ), it is seen to be modified to  $-29.2$  MeV. The increase in the magnitude of the mass-shift, with density  $\rho_B$ , is because of the large drop in the dilaton field  $\chi$  at higher densities. However, with increase in the isospin asymmetry of the medium the magnitude of the mass-shift decreases because the drop in the dilaton field  $\chi$  is less at a higher value of the isospin asymmetry parameter  $\eta$ . For the nuclear matter saturation density  $\rho_B = \rho_0$  and at temperature  $T = 0$ , the mass-shift for  $\psi(3686)$  is observed to be about  $-117$  and  $-114$  MeV for  $\eta = 0$  and  $0.5$  respectively, and for  $\psi(3770)$ , the values of the mass-shift are seen to be about  $-155$  MeV and  $-150$  MeV respectively. At  $\rho_B = 4\rho_0$  and zero temperature, the values of the mass-shift for  $\psi(3686)$  are modified to  $-436$  MeV and  $-396$  MeV for  $\eta = 0$  and  $0.5$  respectively, and, for  $\psi(3770)$ , the drop in the masses are about  $-577$  MeV and  $-523$  MeV respectively. As mentioned earlier, the drop in the dilaton field,  $\chi$ , at finite temperature is less than at zero temperature and this behaviour is reflected in the smaller mass-shift of the charmonium states at finite temperatures as compared to zero temperature case. At nuclear matter saturation density  $\rho_B = \rho_0$ , temperature  $T = 100$  MeV, the values of the mass-shift for the  $J/\psi$  meson are observed to be about  $-6.77$  MeV and  $-6.81$  MeV for isospin symmetric ( $\eta = 0$ ) and isospin asymmetric ( $\eta = 0.5$ ) nuclear medium respectively. At baryon density  $\rho_B = 4\rho_0$ , temperature  $T = 100$  MeV, the mass-shift for  $J/\psi$  is observed to be  $-28.4$  MeV and  $-27.2$  MeV for isospin symmetric ( $\eta = 0$ ) and isospin asymmetric ( $\eta = 0.5$ ) nuclear medium respectively. For the excited charmonium states  $\psi(3686)$  and  $\psi(3770)$ , the mass-shifts at nuclear matter saturation density  $\rho_B = \rho_0$  and temperature  $T = 100$  MeV, are observed to be  $-91.8$  MeV and  $-121.4$  MeV respectively, for isospin symmetric nuclear medium ( $\eta = 0$ ) and  $-92.4$  MeV and  $-122$  MeV for the isospin asymmetric nuclear medium with  $\eta = 0.5$ . For a baryon density of  $\rho_B = 4\rho_0$ , and temperature  $T = 100$  MeV, the mass-shifts for charmonium states  $\psi(3686)$  and ( $\psi(3770)$ ) are

seen to be  $-386$  MeV and  $-510$  MeV respectively, for isospin symmetric ( $\eta = 0$ ) and  $-369$  MeV and  $-488$  MeV for isospin asymmetric nuclear medium with  $\eta = 0.5$ . For temperature  $T = 150$  MeV and at the nuclear matter saturation density  $\rho_B = \rho_0$ , the mass-shifts for the charmonium states  $J/\psi, \psi(3686)$  and  $\psi(3770)$  are seen to be  $-6.25$ ,  $-85$  and  $-112$  MeV respectively in the isospin symmetric nuclear medium ( $\eta = 0$ ). These values are modified to  $-7.2$ ,  $-98$  and  $-129$  MeV respectively, in the isospin asymmetric nuclear medium with  $\eta = 0.5$ . At a baryon density  $\rho_B = 4\rho_0$ , the values of the mass-shift for  $J/\psi, \psi(3686)$  and  $\psi(3770)$  are observed to be  $-26.4$ ,  $-358$  and  $-473$  MeV in isospin symmetric nuclear medium ( $\eta = 0$ ) and in isospin asymmetric nuclear medium with  $\eta = 0.5$ , these values are modified to  $-27.6$ ,  $-375$  and  $-494$  MeV respectively. Note that at high temperatures, e.g., at  $T = 150$  MeV the mass-shift in isospin asymmetric nuclear medium ( $\eta = 0.5$ ) is more as compared to isospin symmetric nuclear medium ( $\eta = 0$ ). This is opposite to what is observed for the zero temperature case. The reason is that at high temperatures the dilaton field  $\chi$  has a larger drop in the isospin asymmetric nuclear medium ( $\eta = 0.5$ ) as compared to the isospin symmetric nuclear medium ( $\eta = 0$ ), due to the contributions from the  $\delta$  field for the nonzero  $\eta$ , which is observed to decrease in its magnitude at high temperatures. The dependence of the wave function of the charmonium on the density and temperature of the medium can be introduced through the modification of the strength of the harmonic potential for the charmonium state [33] given as the parameter  $\beta$  in equation (30). In the medium, one expects the strength of the confining potential to be smaller than in the vacuum as due to medium modifications of the hadrons, more decay channels can become accessible, which are not available in the vacuum. We observe that when the parameter is decreased by about 5%, the mass drops of the charmonium states are increased by about 14% and a change of the parameter  $\beta$  by 2% leads to a mass drop of the charmonium states larger by about 5%.

The values of the mass-shift for the charmonium states obtained within the present investigation, at nuclear matter saturation density  $\rho_0$  and temperature  $T = 0$ , are in good agreement with the mass shift of  $J/\psi, \psi(3686)$  and  $\psi(3770)$  as  $-8$ ,  $-100$  and  $-140$  MeV respectively, at the nuclear matter saturation density computed in Ref. [31] from the second order stark effect, with the gluon condensate in the nuclear medium computed in the

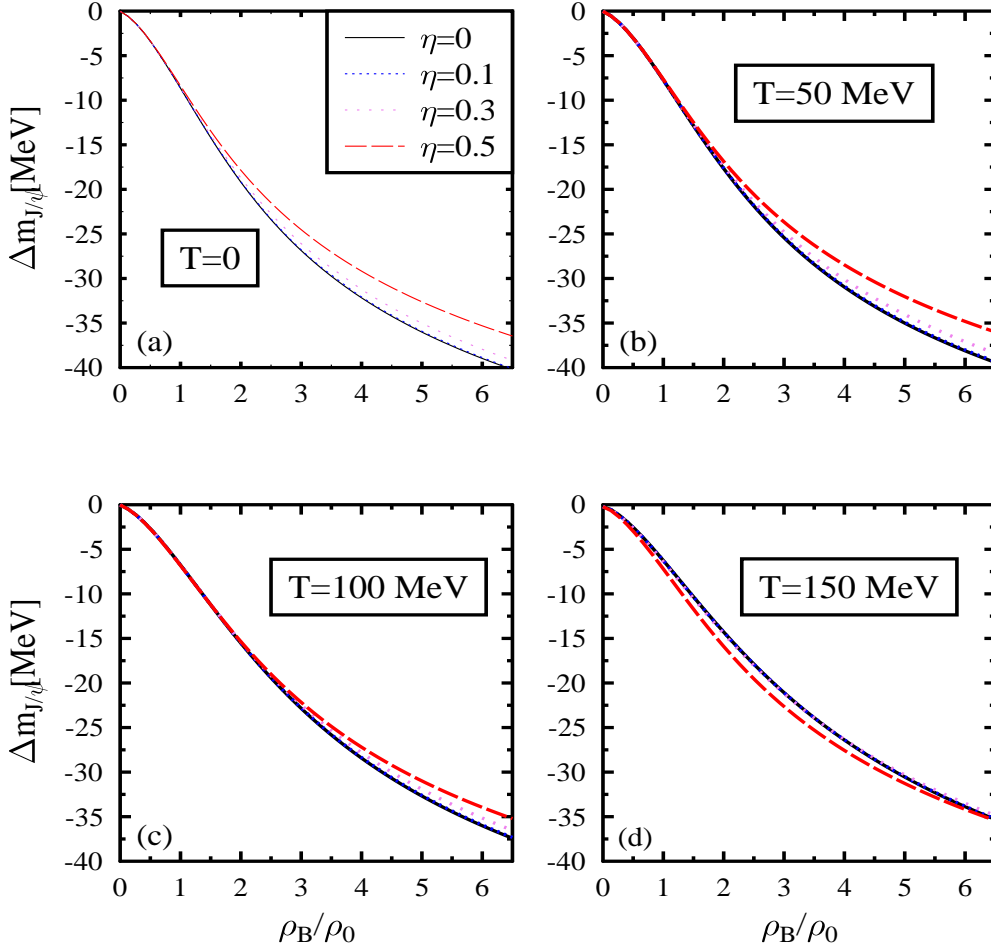


FIG. 15: (Color online) The mass shift of  $J/\psi$  plotted as a function of the baryon density in units of nuclear matter saturation density at given temperature, for different values of the isospin asymmetry parameter,  $\eta$ .

linear density approximation. On the other hand, in the present work, the temperature and density dependence of the gluon condensates are calculated from the medium modifications of the dilaton field within the chiral SU(3) model. In Ref. [31], the masses of the charmonium states were calculated for the symmetric nuclear matter at zero temperature, whereas



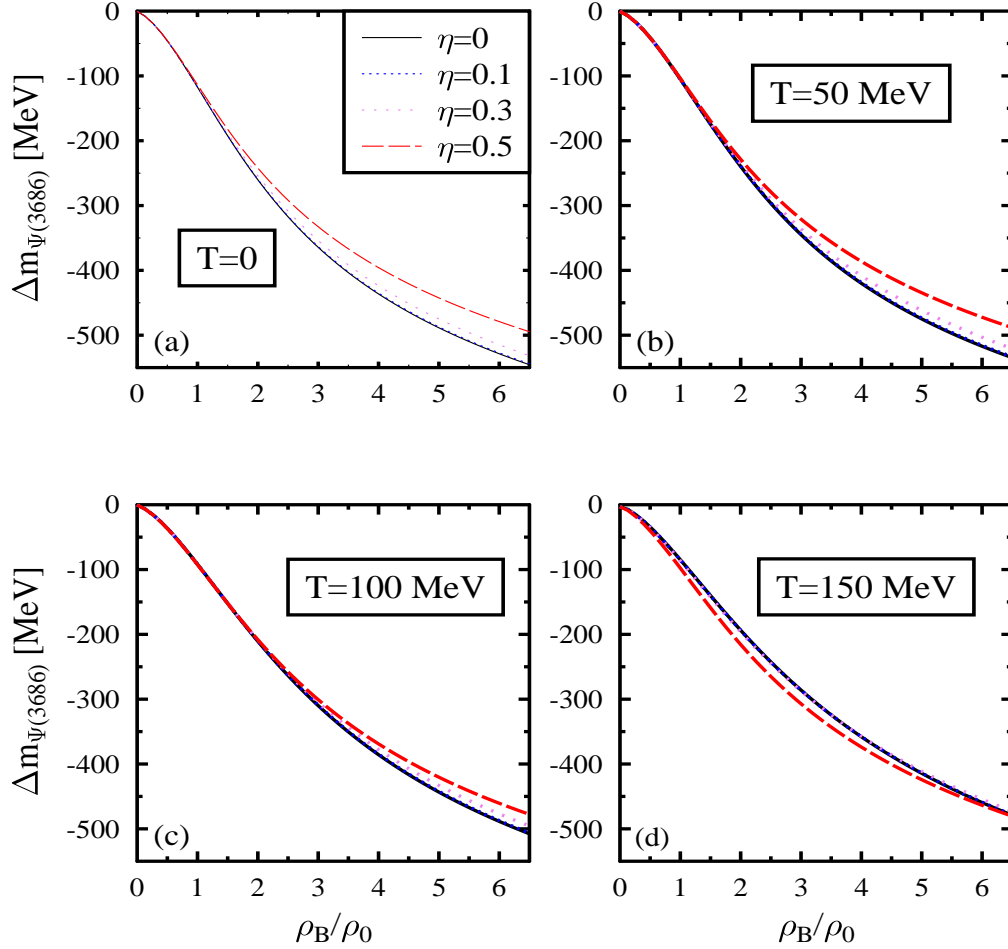


FIG. 16: (Color online) The mass shift of  $\psi(3686)$  plotted as a function of the baryon density in units of nuclear matter saturation density at given temperature, for different values of the isospin asymmetry parameter,  $\eta$ .

the present investigation studies the isospin asymmetry dependence of the masses of the charmonium states in the nuclear medium at finite temperatures, which will be relevant for the asymmetric heavy ion collision experiments planned at the future facility at GSI. The mass-shift for  $J/\psi$  has also been studied with the QCD sum rules in [35] and the value at

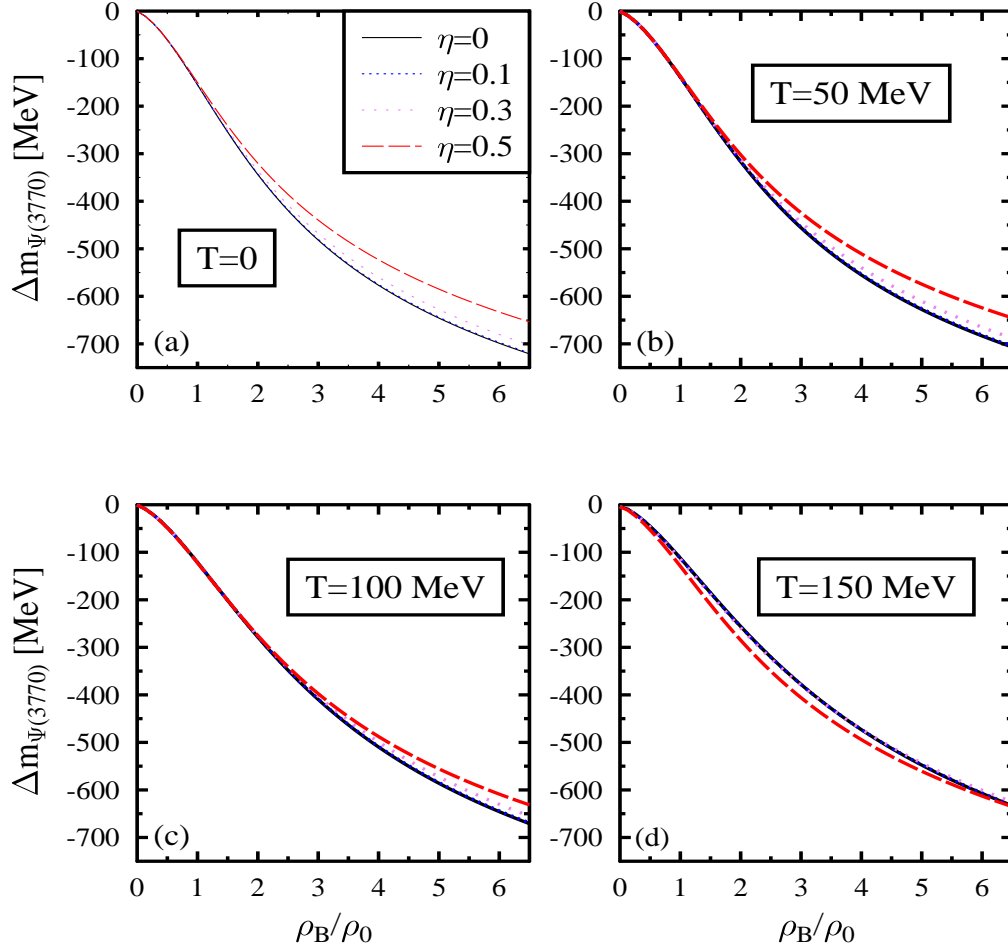


FIG. 17: (Color online) The mass shift of  $\psi(3770)$  plotted as a function of the baryon density in units of nuclear matter saturation density at given temperature, for different values of the isospin asymmetry parameter,  $\eta$ .

nuclear saturation density was observed to be about  $-7$  MeV. In [34] the operator product expansion was carried out upto dimension six and mass shift for  $J/\psi$  was calculated to be  $-4$  MeV at nuclear matter saturation density  $\rho_0$  and at zero temperature. The effect of temperature on the  $J/\psi$  in deconfinement phase was studied in [75, 80]. In these investi-

gations, it was reported that  $J/\psi$  mass remains essentially constant within a wide range of temperature and above a particular value of the temperature,  $T$ , there is seen to be a sharp change in the mass of  $J/\psi$  in the deconfined phase. For example, in Ref. [81] the mass shift for  $J/\psi$  was reported to be about 200 MeV at  $T = 1.05 T_c$ . In the present work, we have studied the effects of temperature, density and isospin asymmetry, on the mass modifications of the charmonium states ( $J/\psi$ ,  $\psi(3686)$  and  $\psi(3770)$ ) in the confined hadronic phase, arising due to modifications of a scalar dilaton field which simulates the gluon condensates of QCD, within a chiral SU(3) model. The effect of temperature is found to be small for the charmonium states  $J/\psi(3097)$ ,  $\psi(3686)$  and  $\psi(3770)$ , whereas the masses of charmonium states are observed to vary considerably with density, in the present investigation.

The medium modifications of the masses of  $D$  and  $\bar{D}$  mesons as well as that of charmonium states could be an explanation for the observed  $J/\psi$  suppression observed by NA50 collaboration at 158 GeV/nucleon in the Pb-Pb collisions [20]. Due to the drop in the mass of the  $D\bar{D}$  pair in the nuclear medium, it can become a possibility that the excited states of charmonium ( $\psi'$ ,  $\chi_{c2}$ ,  $\chi_{c1}$ ,  $\chi_{c0}$ ) can decay to  $D\bar{D}$  pairs [48] and hence the production of  $J/\Psi$  from the decay of these excited states can be suppressed. Even at high values of densities at given temperatures, it can become a possibility that  $J/\psi$  itself decays to  $D\bar{D}$  pairs. Thus the medium modifications of the  $D$  mesons can modify the decay widths of the charmonium states [33]. In figures 18 and 19, we show the density dependence of the masses of the  $D^+D^-$  as well as  $D^0\bar{D}^0$  pairs calculated in the present investigation, for temperatures,  $T = 0, 100$  and  $150$  MeV and for isospin asymmetry parameter,  $\eta = 0$  and  $\eta=0.5$  respectively. We also show the in-medium masses of the charmonium states  $J/\psi$ ,  $\psi(3686)$  and  $\psi(3770)$  in these figures. We observe that in the isospin symmetric nuclear medium at zero temperature, the in-medium mass of charmonium  $\psi(3770)$  is less than  $D^+D^-$  and  $D^0\bar{D}^0$  pairs above baryon densities  $0.6\rho_0$  and  $0.8\rho_0$  respectively and therefore its decay does not seem possible at densities higher than these densities in the nuclear medium. However, as we move to the isospin asymmetric medium ( $\eta = 0.5$ ), the medium modifications for the masses of the  $D\bar{D}$  pairs as well as of the charmonium states indicate that the decay of  $\psi(3770)$  to  $D^+D^-$  pairs can be possible above a density of about  $2\rho_0$ , but the decay to  $D^0\bar{D}^0$  does not seem possible above a density of about nuclear matter saturation density. In the isospin

symmetric nuclear medium, the decay of the charmonium state  $\psi(3686)$  to  $D^+D^-$  and to  $D^0\bar{D}^0$  seem possible above densities of about  $3.4 \rho_0$  and  $3.3 \rho_0$  respectively. In isospin asymmetric nuclear medium ( $\eta = 0.5$ ) the decay of the charmonium state  $\psi(3686)$  to  $D^+D^-$  pairs seem as possibilities above a density of about  $2\rho_0$ . The effects of the temperatures on the decay of the charmonium states to  $D\bar{D}$  pairs have also been illustrated in the figures 18 and 19. The decay of  $\psi(3770)$  to the  $D\bar{D}$  pairs do not seem possible above a density of about  $\rho_0$  even at  $T=100$  and  $150$  MeV. However, for  $\psi(3686)$ , for  $T=100$  MeV, the decay to the  $D^+D^-$  and  $D^0\bar{D}^0$  seem possible above densities of about  $4\rho_0$  and  $3.8 \rho_0$  for symmetric nuclear matter. For  $T=150$  MeV, these values are modified to  $4.5\rho_0$  and  $4.3 \rho_0$  respectively, for  $\eta=0$ . As we move to the asymmetric nuclear matter, the densities above which the decay of  $\psi(3686)$  decaying to  $D^+D^-$  becomes possible are  $2.4 \rho_0$  and  $2.2 \rho_0$  respectively. Similar to the zero temperature case, we do not see a possibility of  $\psi(3686)$  to  $D^0\bar{D}^0$  for the asymmetric nuclear matter ( $\eta=0.5$ ) at  $T=100$  and  $150$  MeV. In the present investigation, the decay of  $J/\psi$  to  $D\bar{D}$  pairs does not seem as a possibility even upto a density of about  $6\rho_0$ . We observe from figures 18 and 19 that the temperature dependence is minimal for the decay of the charmonium states to the  $D\bar{D}$  pairs even though the density dependence is quite appreciable.

The decay of the charmonium states have been studied in Ref. [29, 33]. It is seen to depend sensitively on the relative momentum in the final state. These excited states might become narrow [33] though the  $D$  meson mass is decreased appreciably at high densities. It may even vanish at certain momentum corresponding to nodes in the wave function [33]. Though the decay widths for these excited states can be modified by their wave functions, the partial decay width of  $\chi_{c2}$ , owing to absence of any nodes, can increase monotonically with the drop of the  $D^+D^-$  pair mass in the medium. This can give rise to depletion in the  $J/\Psi$  yield in heavy-ion collisions. The dissociation of the quarkonium states ( $\Psi', \chi_c, J/\Psi$ ) into  $D\bar{D}$  pairs has also been studied [82, 83] by comparing their binding energies with the lattice results on the temperature dependence of the heavy-quark effective potential [84].

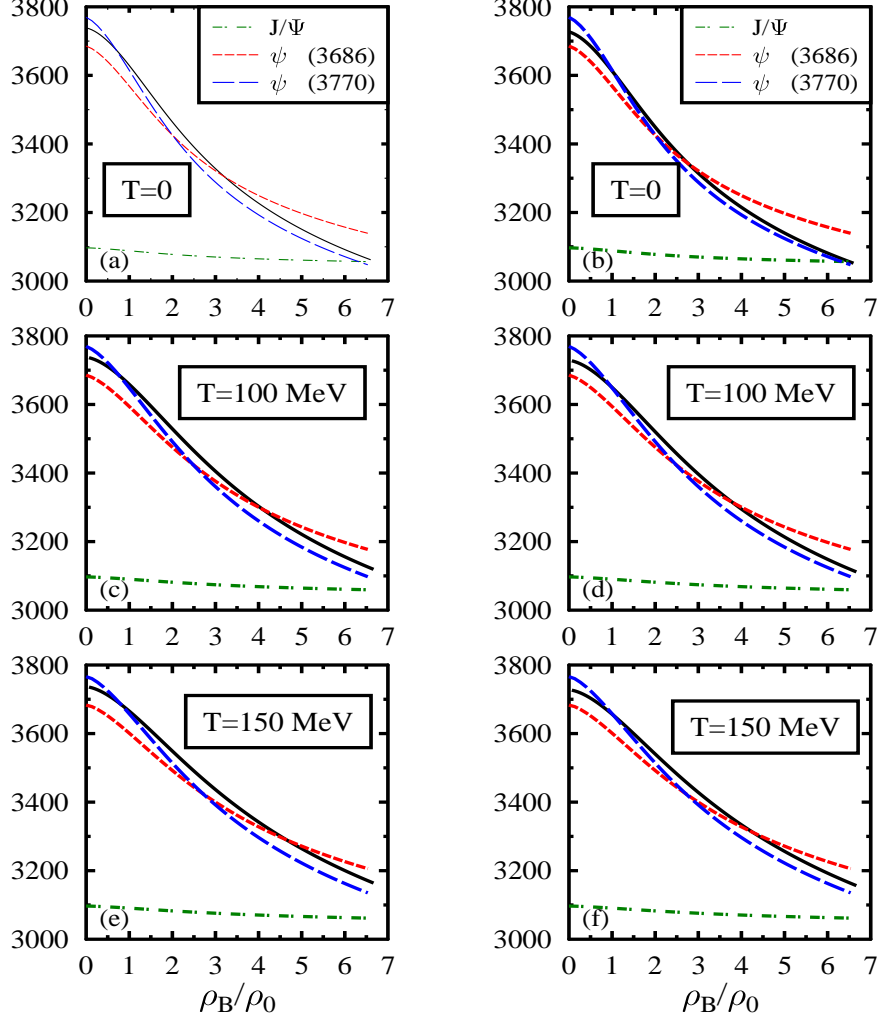


FIG. 18: (Color online) The masses of the  $DD\bar{D}$  pairs [ $D^+D^-$  in (a), (c), (e) and  $D^0\bar{D}^0$  in (b), (d), (f)] in MeV plotted as functions of  $\rho_B/\rho_0$  for isospin for the symmetric nuclear matter ( $\eta=0$ ) and for temperatures,  $T = 0, 100, 150$  MeV.

## VI. SUMMARY

We have investigated in a chiral model the in-medium masses of the  $D$ ,  $\bar{D}$  mesons and the charmonium states ( $J/\psi$ ,  $\psi(3686)$  and  $\psi(3770)$ ) in hot isospin asymmetric nuclear matter.

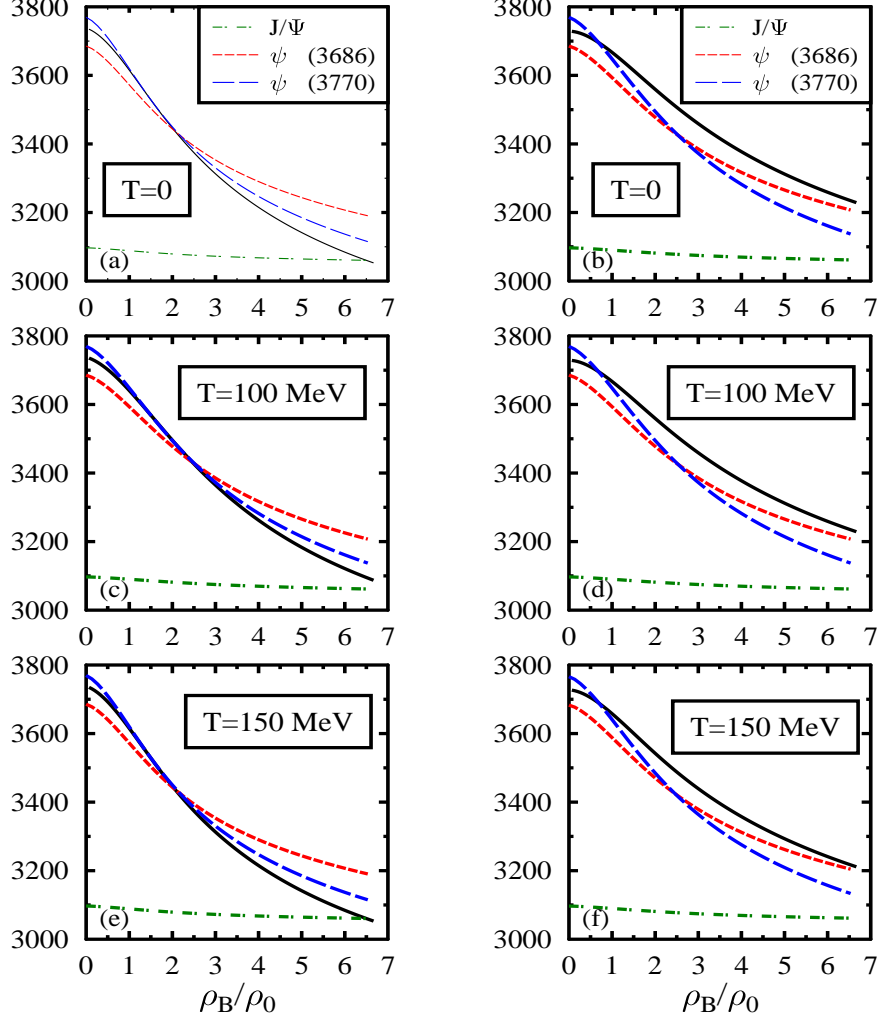


FIG. 19: (Color online) The masses of the  $D\bar{D}$  pairs [ $D^+D^-$  in (a), (c), (e) and  $D^0\bar{D}^0$  in (b), (d), (f)] in MeV plotted as functions of  $\rho_B/\rho_0$  for isospin asymmetry parameter value  $\eta = 0.5$  and temperatures,  $T = 0, 100, 150$  MeV.

The properties of the light hadrons – as studied in  $SU(3)$  chiral model – modify the  $D(\bar{D})$  meson properties in the dense and hot hadronic matter. The  $SU(3)$  model, with parameters fixed from the properties of the hadron masses in vacuum and low-energy KN scattering data, is extended to  $SU(4)$  to derive the interactions of  $D(\bar{D})$  mesons with the light hadron

sector. The mass modifications of  $D^+$  and  $D^0$  mesons is strongly dependent on isospin-asymmetry of medium when we determine the parameters  $d_1$  and  $d_2$  consistent with the KN scattering lengths. However, the sensitivity to the isospin asymmetry is seen to be more for the  $\bar{D}$  doublet, when we fit the parameters to the DN scattering lengths as calculated in the coupled channel approach in Ref. [56]. At finite densities, the masses of  $D(\bar{D})$  mesons are observed to increase with temperature [47] upto a temperature above which it is observed to decrease. The mass modification for the  $D$  mesons are seen to be similar to earlier finite density calculations of QCD sum rules [38, 74] as well as to the quark-meson coupling model [39]. This is in contrast to the small mass modifications in the coupled channel approach [51, 55]. Also we obtain small attractive mass shifts for  $\bar{D}$  mesons similar to the results obtained from the QMC model, which might lead to formation of charmed mesic nuclei. These results for the  $\bar{D}$  mesons are contrary to the results from the coupled channel approach [55], where the  $\bar{D}$  mesons experience a repulsive interaction in the nuclear medium. In our calculations the presence of the repulsive first range term (with coefficient  $-\frac{1}{f_D}$  in Eq. (18)) is compensated by the attractive  $d_1$  and  $d_2$  terms in Eq.(18). Among the attractive range terms ( $d_1$  and  $d_2$  terms),  $d_1$  term is found to be dominating over  $d_2$  term.

We have investigated in the present work, the effects of density, temperature and isospin asymmetry of the nuclear medium on the masses of the charmonium states  $J/\psi$ ,  $\psi(3686)$  and  $\psi(3770)$ , arising due to modification of the scalar dilaton field,  $\chi$ , which simulates the gluon condensates of QCD, within the chiral SU(3) model. The change in the mass of  $J/\psi$  with the density is observed to be small at nuclear matter saturation density and is in agreement with the QCD sum rule calculations. There is seen to be appreciable drop in the in-medium masses of excited charmonium states  $\psi(3686)$  and  $\psi(3770)$  with density. The mass drop of the excited charmonium states  $\psi(3686)$  and  $\psi(3770)$  are large enough to be seen in the dilepton spectra emitted from their decays in experiments involving  $\bar{p}$ -A annihilation in the future facility at GSI, provided these states decay inside the nucleus. The life time of the  $J/\psi$  has been shown to be almost constant in the nuclear medium, whereas for these excited charmonium states, the life times are shown to reduce to less than 5 fm/c, due to appreciable increase in their decay widths [33]. Hence a significant fraction of the produced excited charmonium states in these experiments are expected to decay inside

the nucleus [85]. The in-medium properties of the excited charmonium states  $\psi(3686)$  and  $\psi(3770)$  can be studied in the dilepton spectra in  $\bar{p}$ -A experiments in the future facility of the FAIR, GSI [16]. The mass shift of the charmonium states in the hot nuclear medium seem to be appreciable at high densities as compared to the temperature effects on these masses, and these should show in observables like the production of these charmonium states in the compressed baryonic matter experiment at the future facility at GSI, where baryonic matter at high densities and moderate temperatures will be produced.

The medium modifications of the  $D$  meson masses can lead to a suppression in the  $J/\Psi$  yield in heavy-ion collisions, since the excited states of the  $J/\Psi$  can decay to  $D\bar{D}$  pairs in the dense hadronic medium. The medium modifications of the masses of the charmonium states as well as the  $D$  and  $\bar{D}$  mesons have been considered in the present investigation. The isospin asymmetry lowers the density at which decay to  $D^+D^-$  pairs occur. Due to increase in the mass of  $D^0\bar{D}^0$  in the isospin-asymmetric medium, isospin-asymmetry is seen to disfavor the decay of the charmonium states to the  $D^0\bar{D}^0$  pairs. At zero or finite temperatures, there does not seem to be a possibility of decay of  $J/\Psi$  to  $D^+D^-$  or  $D^0\bar{D}^0$  pairs. The isospin dependence of  $D^+$  and  $D^0$  masses is seen to be a dominant medium effect at high densities, which might show in their production ( $D^+/D^0$ ), whereas, for the  $D^-$  and  $\bar{D}^0$ , one sees that, even though these have a strong density dependence, their in-medium masses remain similar at a given value for the isospin-asymmetry parameter  $\eta$ . This is the case when we fit the parameters  $d_1$  and  $d_2$  from the KN scattering lengths. When we determine these parameters from the DN scattering lengths as calculated in Ref. [56], the masses of the  $\bar{D}$  doublet are seen to be more sensitive to isospin asymmetry in the medium. The strong density dependence as well as the isospin dependence of the  $D(\bar{D})$  meson optical potentials in asymmetric nuclear matter can be tested in the asymmetric heavy-ion collision experiments at future GSI facility [16] in observables like the  $D^+/D^0$  as well as  $D^-/\bar{D}^0$  ratios. In the present work, we have investigated the in-medium masses of the charmonium states due to their interaction with the scalar dilaton field (simulating the gluon condensates of QCD) and  $D(\bar{D})$  mesons due to the interaction with nucleons as well as scalar mesons. The parameters of the asymmetric nuclear matter are fitted to the nuclear matter properties, vacuum baryon masses, the hyperon potentials as well as the KN scattering lengths in the



chiral SU(3) model. The study of the in-medium modifications of  $D$  mesons in hadronic matter including hyperons along with nucleons at zero and finite temperatures, as well as the study of the medium modifications of the strange open charm mesons can be possible extensions of the present investigation.

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